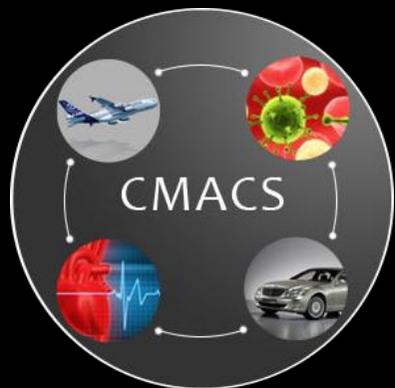


Simulations of Complex Systems using WebGL and HTML5: Exploiting Your Computer's GPUs for Real Time and Platform- Independent Interactive Calculations

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College of Veterinary Medicine, Cornell University, NY

In collaboration with: Elizabeth Cherry, Ezio Bartocci, Scott Smolka, Radu Grosu and Evgeny Demidov

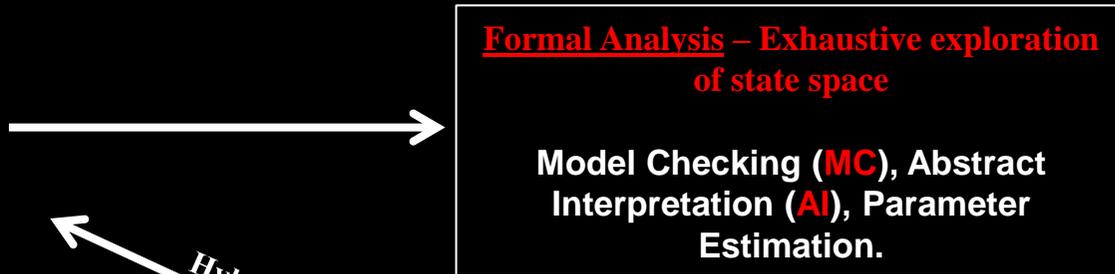


*Spring 2012 CMACS Virtual PI Meeting
April 20 and 27, 2012*

Motivation (abstraction)

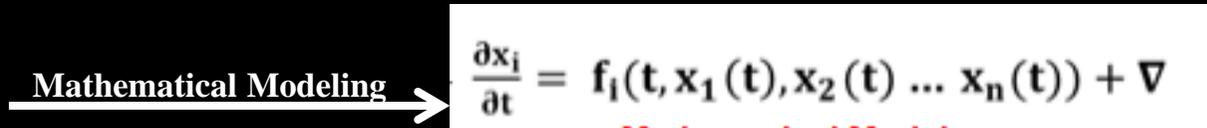
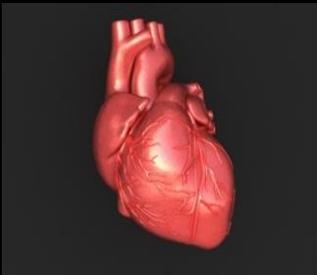


Computational Model
Linear Hybrid
Automata (LHA),
Kripke structure, etc.

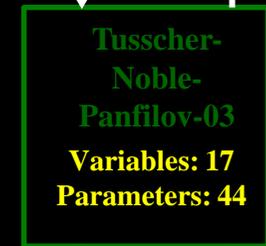
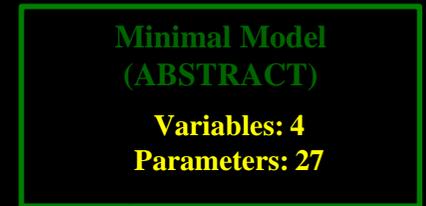


Qualitative/
Quantitative **Insights**
(Abstract parameter
and state-space)

Hybridization, over-
approximation, abstraction



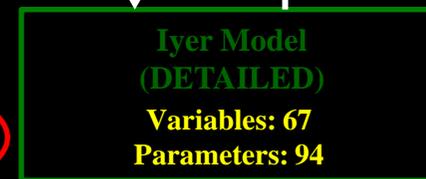
Mathematical Model
(Possibly Non-linear)



Intermediate
Models

Systematic
Refinement

Abstraction

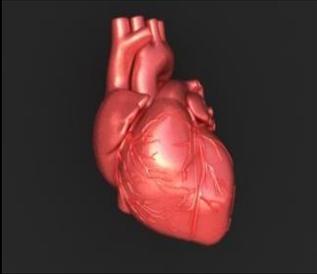


Tower of Abstraction for Cardiac
Models

Biological Phenomena
(Cardiac excitation: cell
& tissue-level behavior)

- Physiological Insights
- Root-cause detection
- Personalized treatment
- Pharmacology

Motivation (Model Checking)



Mathematical Modeling →

$$\frac{\partial x_i}{\partial t} = f_i(t, x_1(t), x_2(t) \dots x_n(t)) + \nabla$$

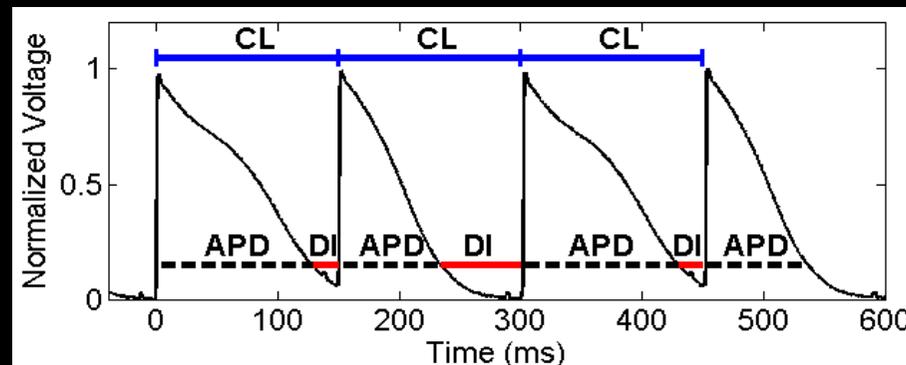
Mathematical Model

Biological Phenomena
(Cardiac excitation: cell
& tissue-level behavior)

Model Checking

Mathematical cell models -----→ normal or disease

- Model of cardiac cell and excitability (useful for example ischemia characterization) 
- Alternans in time and space (temporal and spatial extension of Model checking)



Motivation



During Model Checking and Abstract interpretation of cardiac models
We still need to solve complex and reduced models for comparison

CMACS: We aim to gain fundamental new insights into the emergent behavior of complex biological and embedded systems through the use of revolutionary, highly scalable and fully automated modeling and analysis techniques

Motivation



During Model Checking and Abstract interpretation of cardiac models
We still need to solve complex and reduced models for comparison

CMACS: We aim to gain fundamental new insights into the emergent behavior of complex biological and embedded systems through the use of **revolutionary, highly scalable** and fully automated **modeling** and analysis **techniques**

Modeling with GPU

Motivation



Modeling with GPU

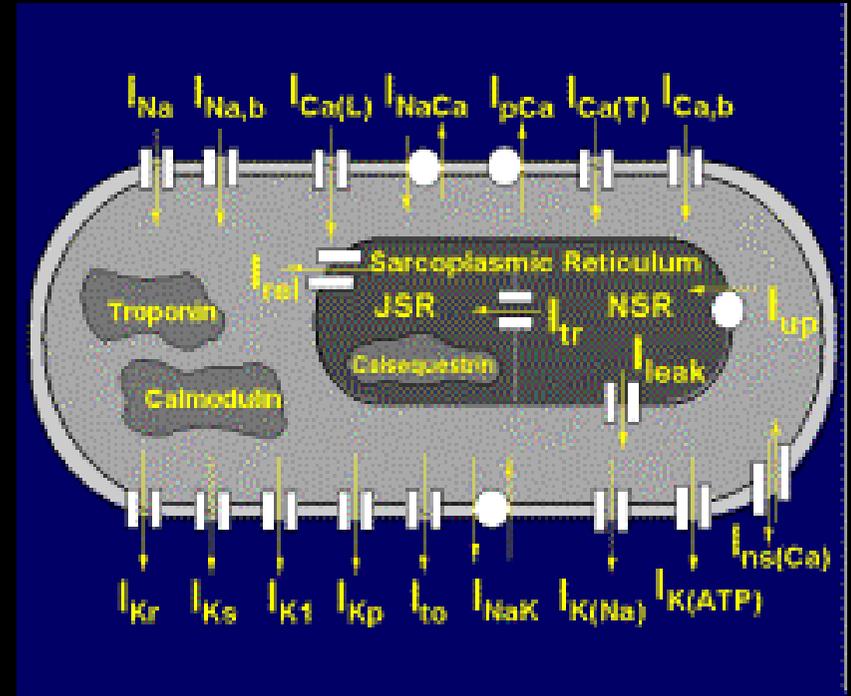
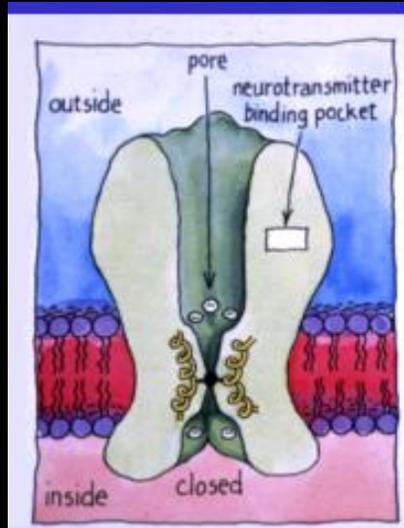
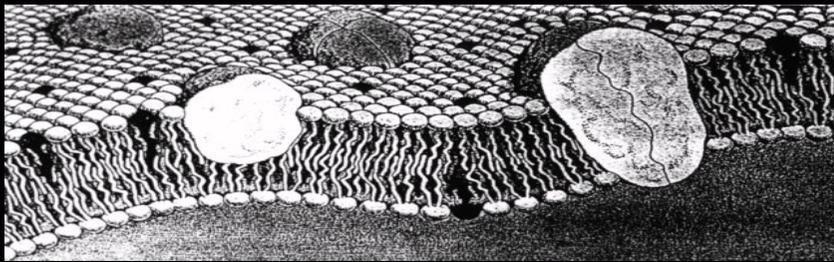
A Graphics Processing Unit (GPU) is a specialized electronic circuit designed to rapidly access and manipulate memory to accelerate the generation of images for fast output to a display.

Over the past decade due to its massively parallel architecture structure it has become a mean to accelerate general purpose scientific and engineering computing.

WebGL is a new web-based cross-platform technology that allows the execution of JavaScript and Shader codes directly to a computer's GPU from a web browser without the need for any plug-ins.

Therefore it is now possible to run high-performance parallel computing simulations over the web on a local PC or laptop independent of the operating system used.

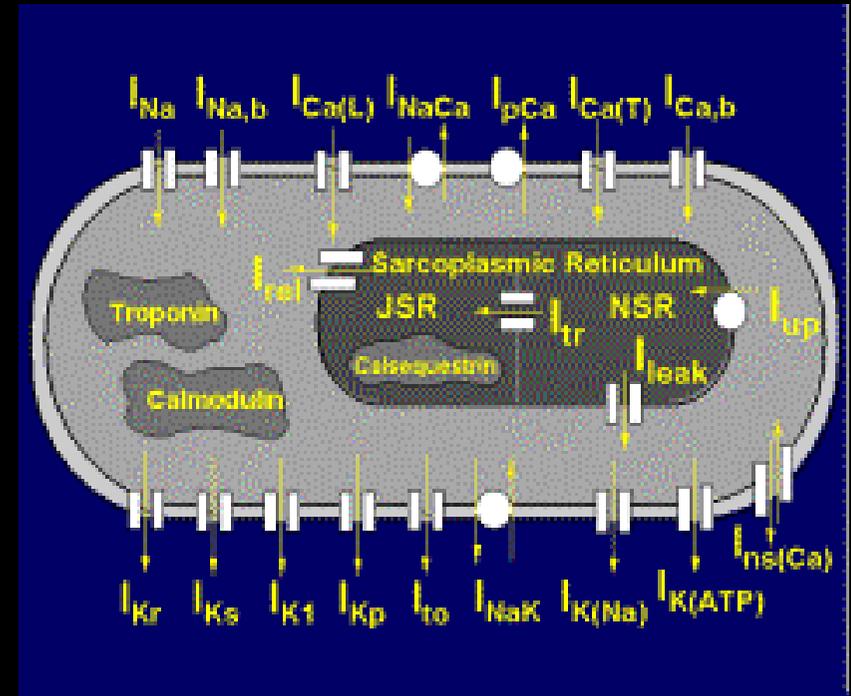
Membrane Potential Models



Membrane Potential Models

About 92 models developed so far for mammalian cardiac cells

- Noble model. 1962; 4 ODEs
(Generic Purkinje myocyte)
- Beeler-Reuter model. 1977; 8 ODEs
(Generic ventricle myocyte)
- Hund-Rudy model. 2004; 29 ODEs
(Canine, ventricle, myocyte)
- Iyer et al. model. 2007; 67 ODEs
(Human, ventricle, myocyte)
- Fenton-Karma model. 1998 (2008); 3 (4) ODEs
(minimal model that fits to experiments and other models)

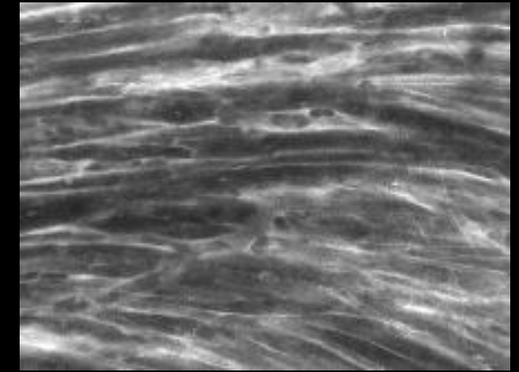
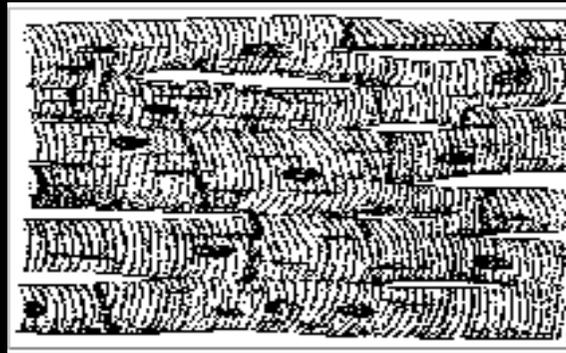
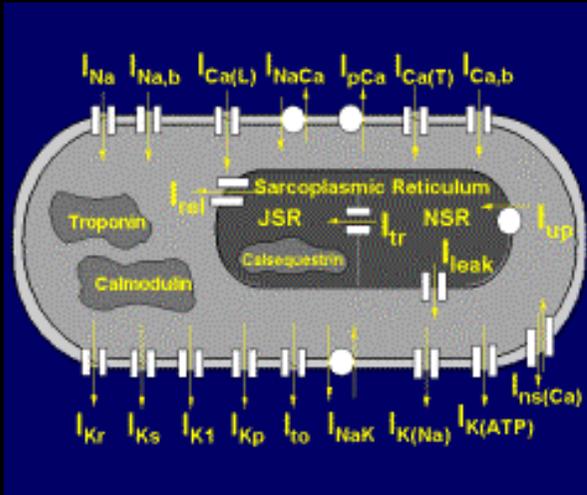


Fenton and Karma (1998), Chaos 8;20-47

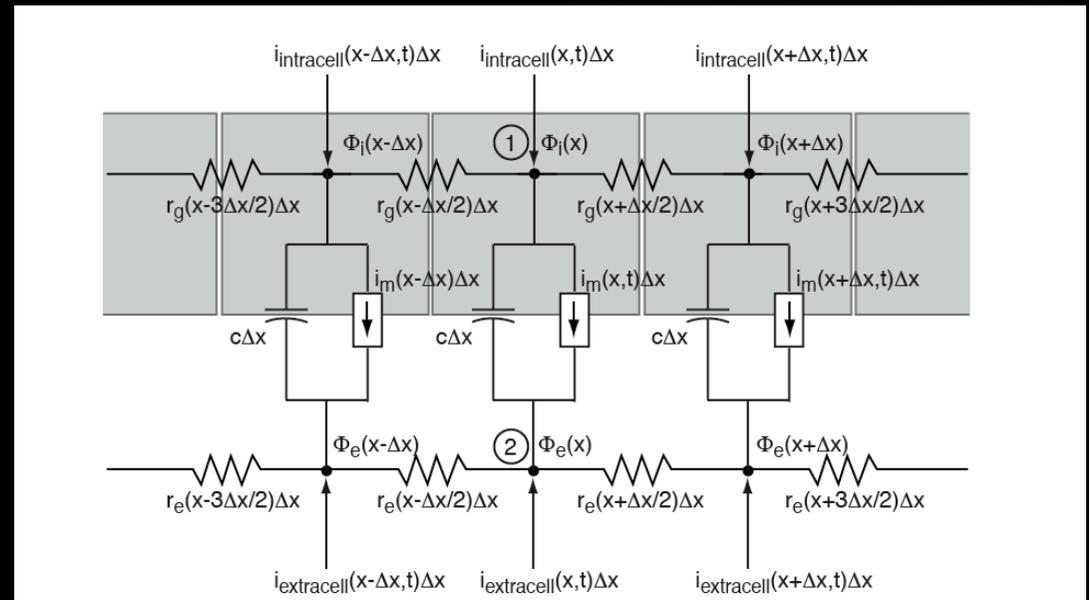
Bueno, Cherry and Fenton (2008), JTB 253; 544-560

Flavio H Fenton and Elizabeth M. Cherry (2008), Scholarpedia, 3(8):1868

Electrical Waves in Tissue



Cardiac tissue is a reaction-diffusion system.



$$V_m = \phi_i - \phi_e,$$

$$\nabla(\sigma_i \nabla \phi_i) = I_m = \beta_{sv} \left(C_m \frac{\partial}{\partial t} V_m + I_{ion} \right),$$

$$\nabla(\sigma_e \nabla \phi_e) = I_m = \beta_{sv} \left(C_m \frac{\partial}{\partial t} V_m - I_{ion} \right),$$

$$\nabla \sigma_b \nabla \phi_b = -I_{stim}.$$

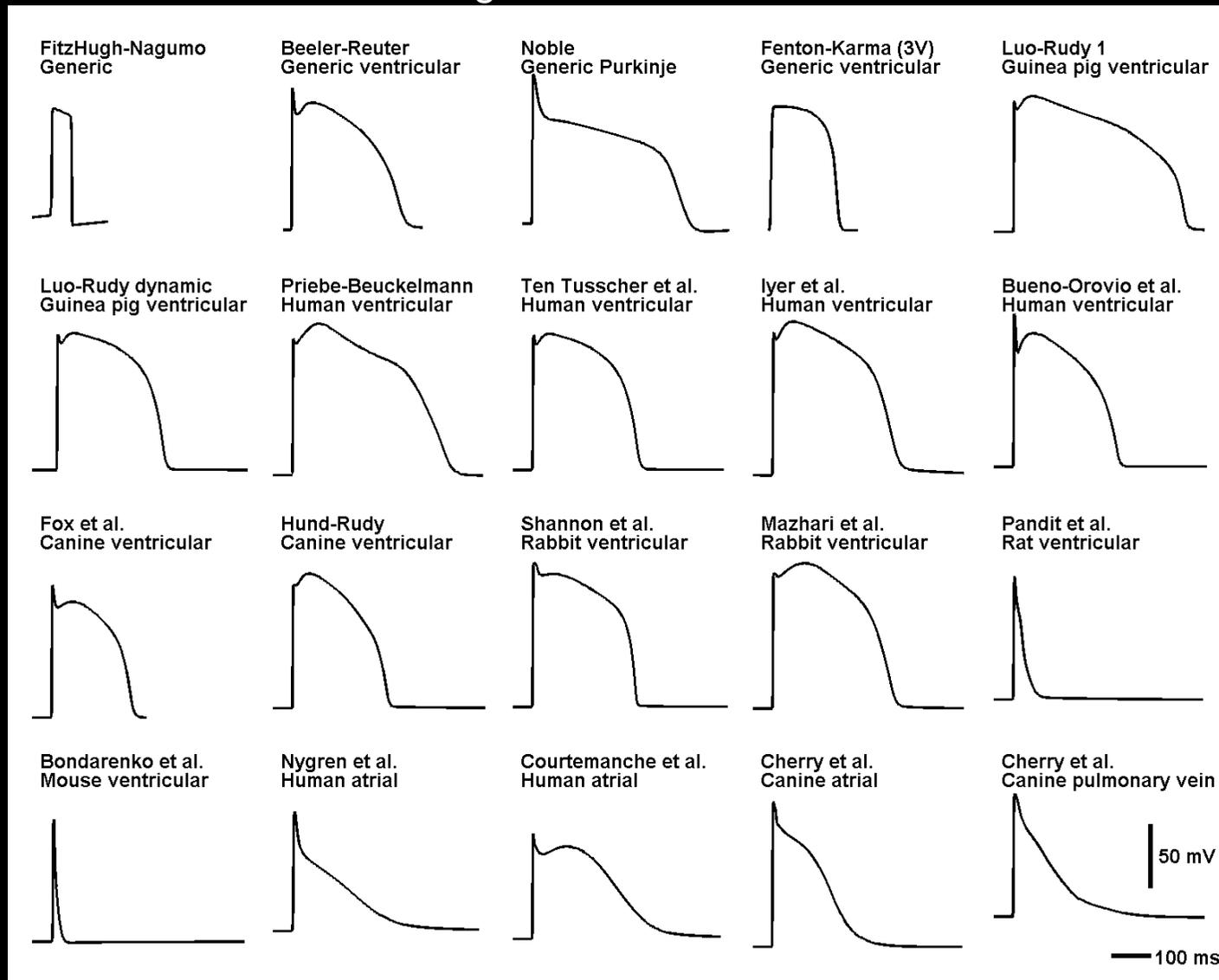
$$n(\sigma_i \nabla \phi_i) = 0,$$

$$n(\sigma_e \nabla \phi_e) = n(\sigma_b \hat{l}) \nabla \phi_i,$$

$$\phi_e = \phi_b,$$

Many models for diverse cell types

Implemented (~40) most of the existing ionic models in single cell and in tissue

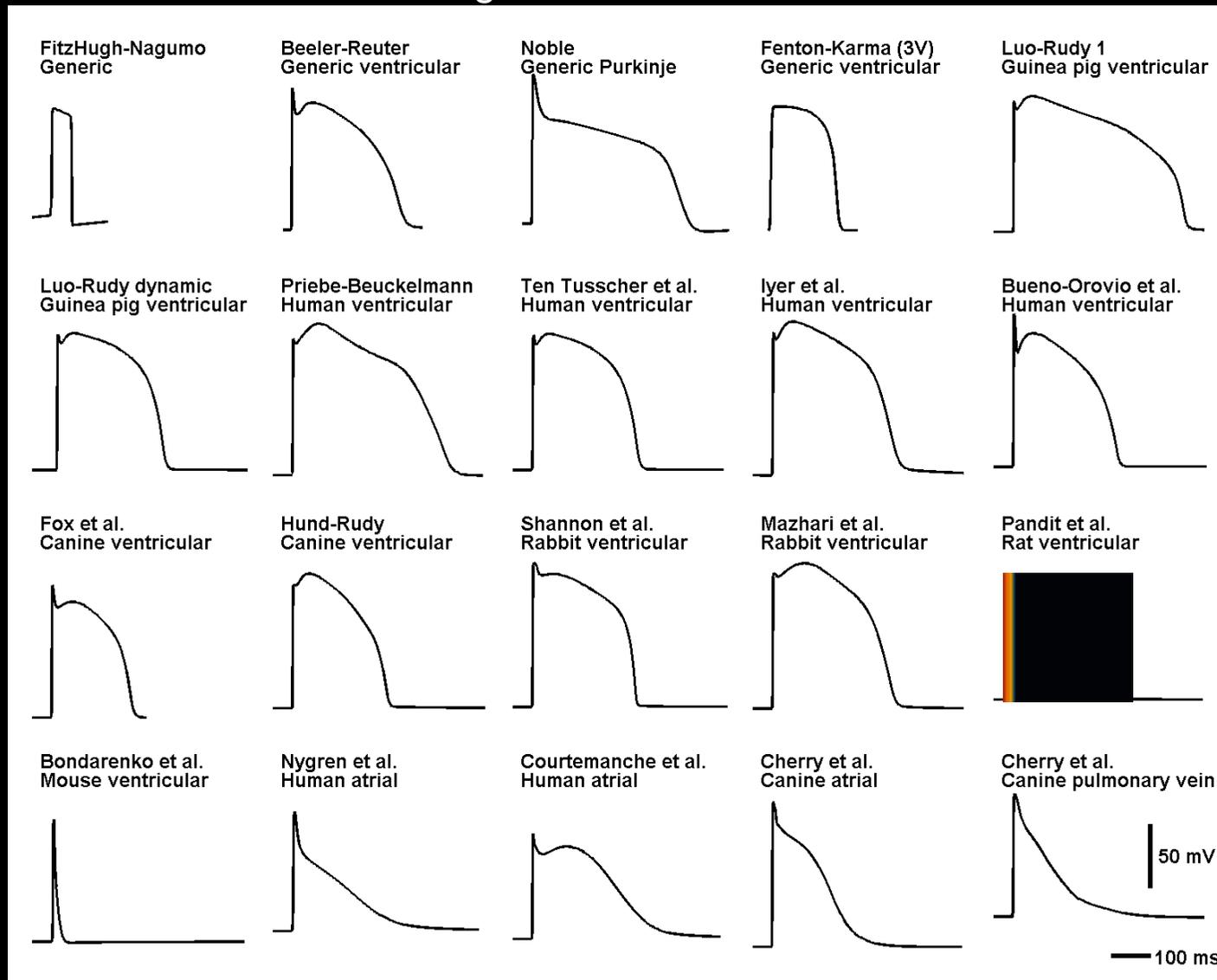


Models of cardiac cell

Flavio H Fenton and Elizabeth M. Cherry (2008), Scholarpedia, 3(8):1868.

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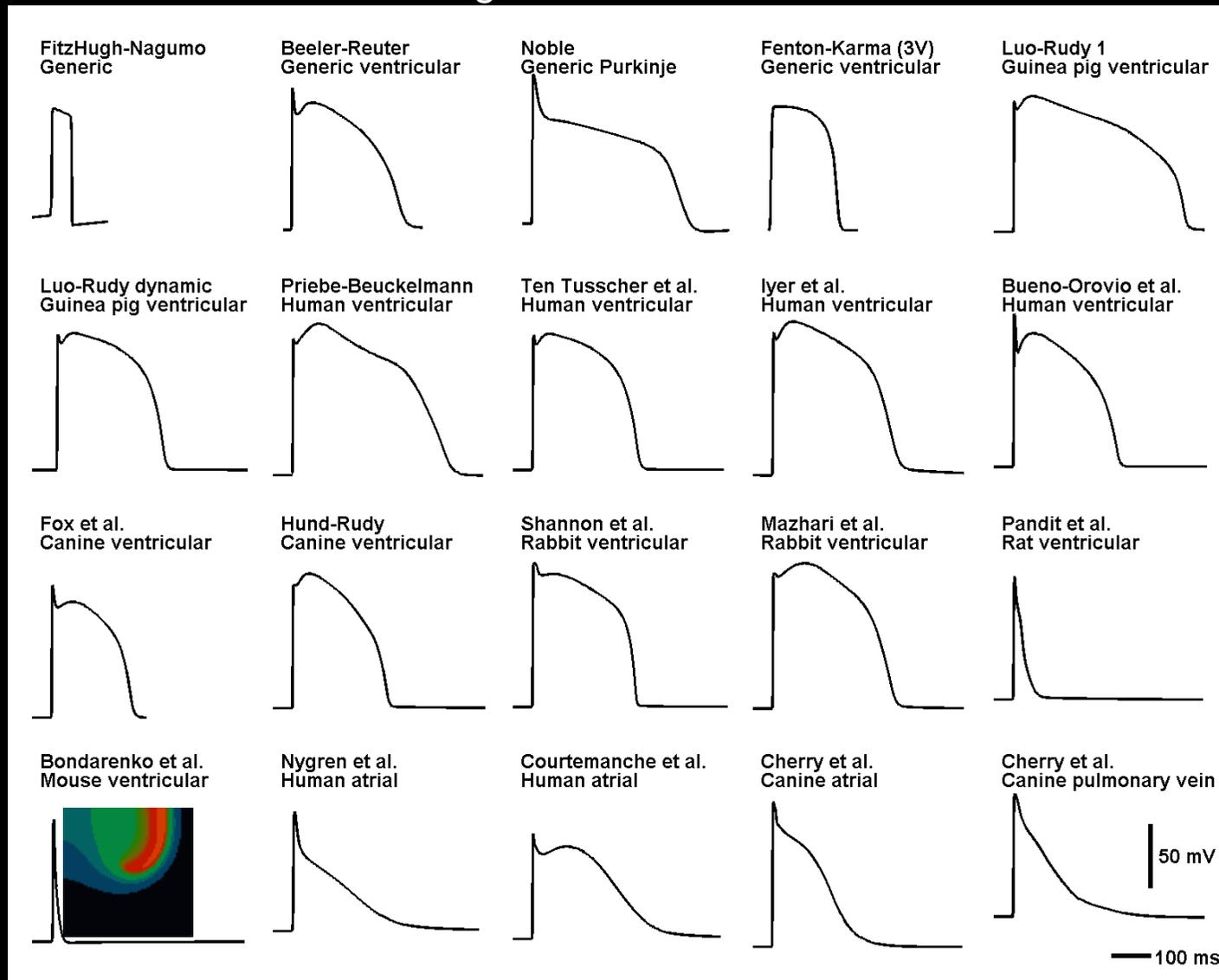


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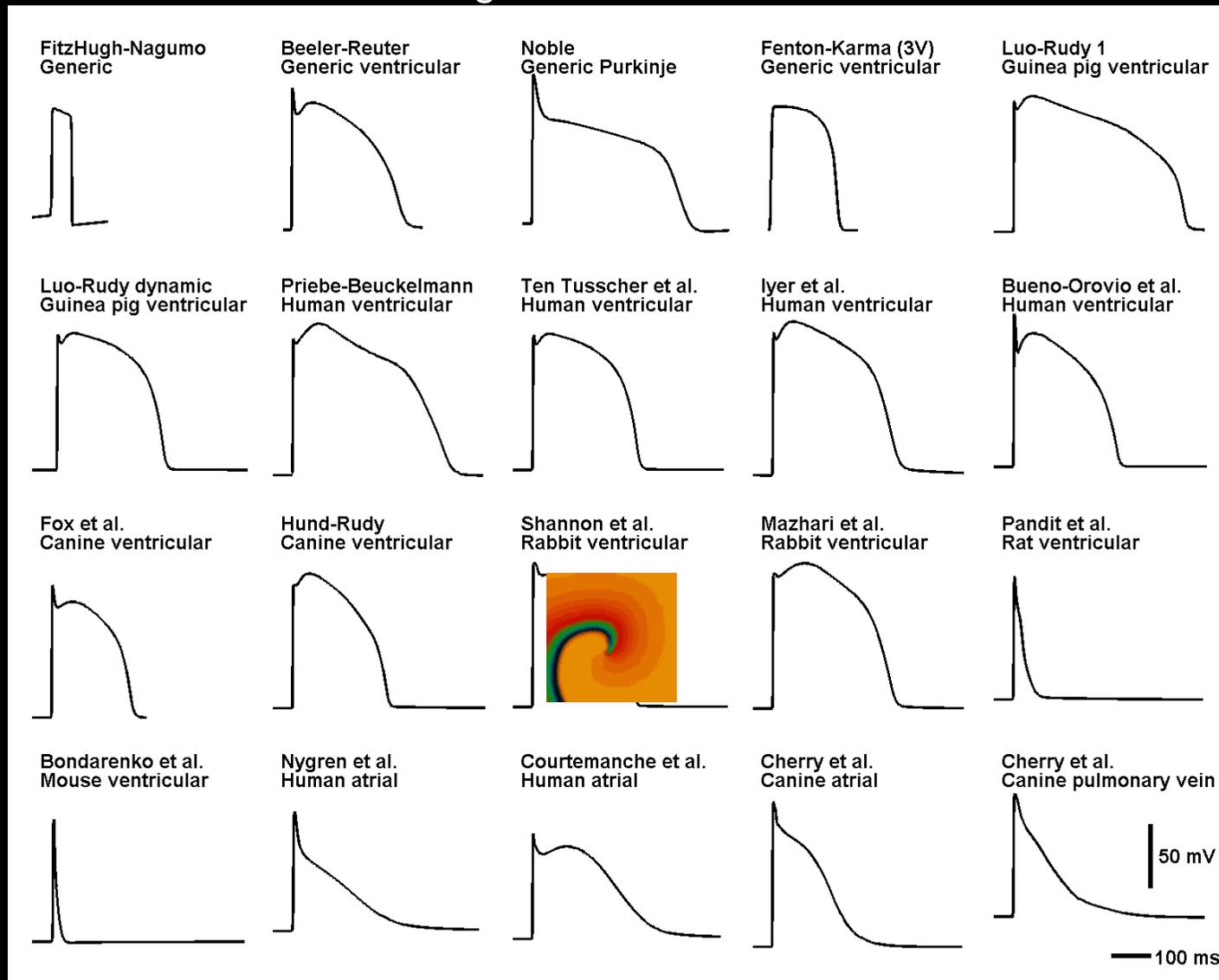


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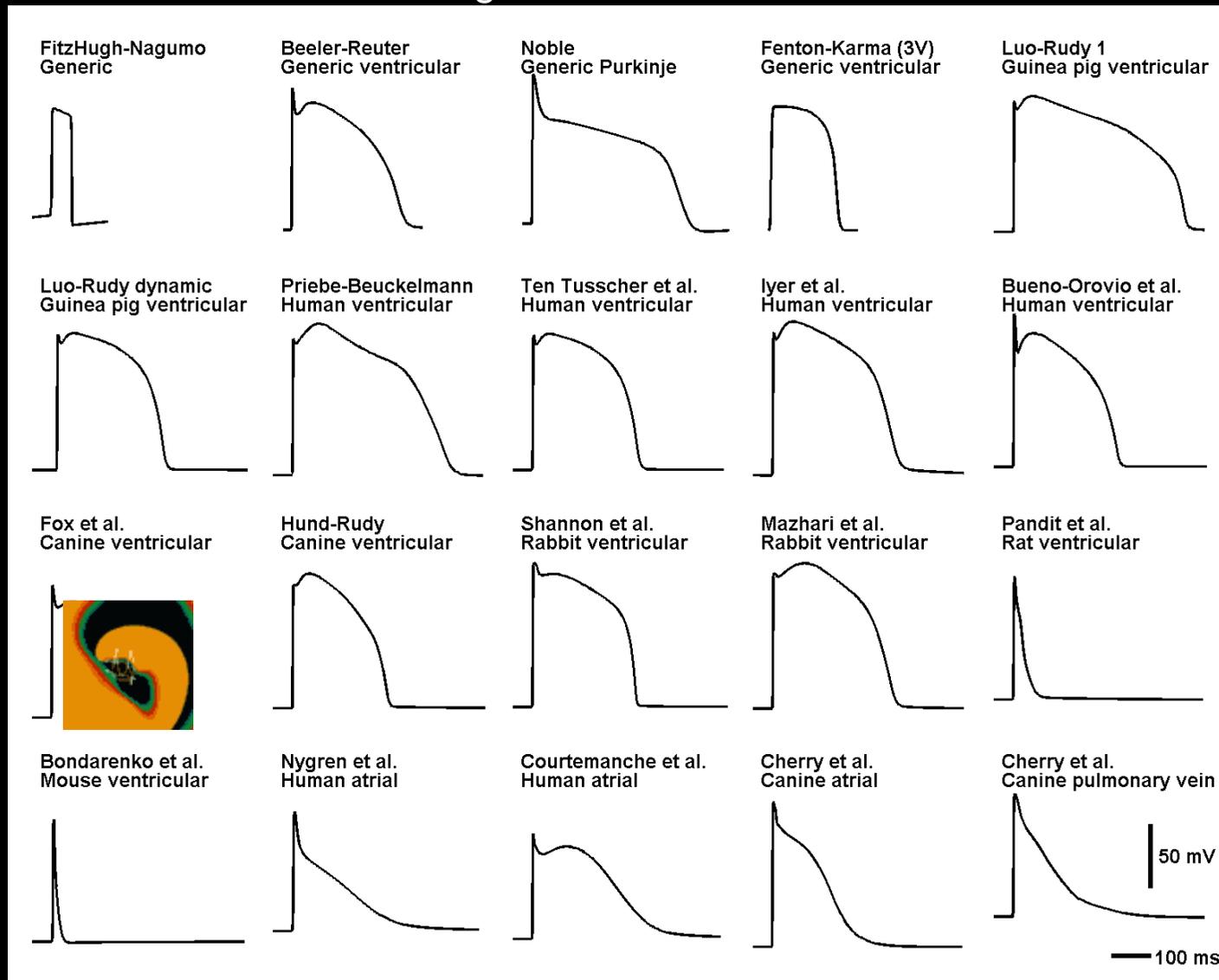


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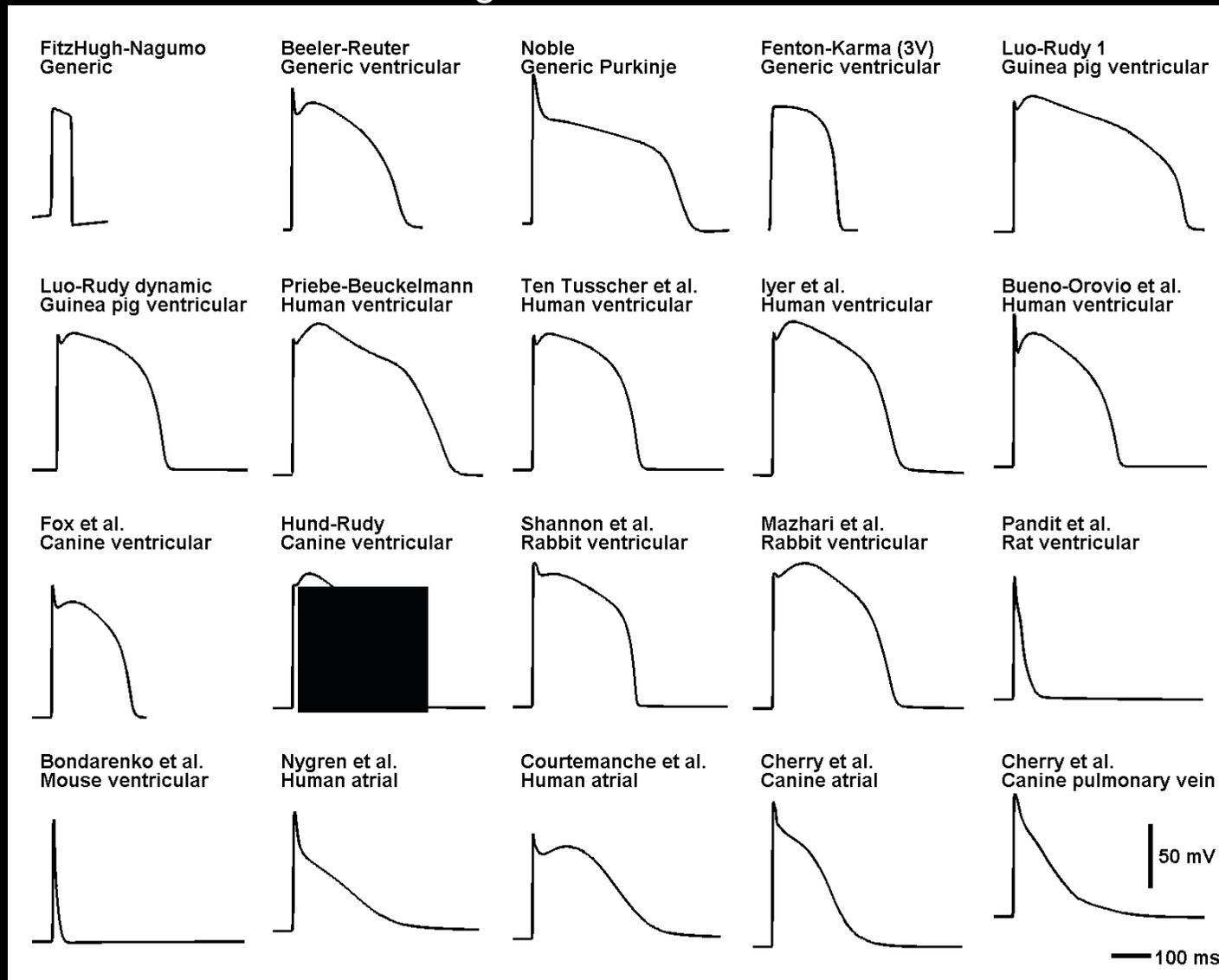


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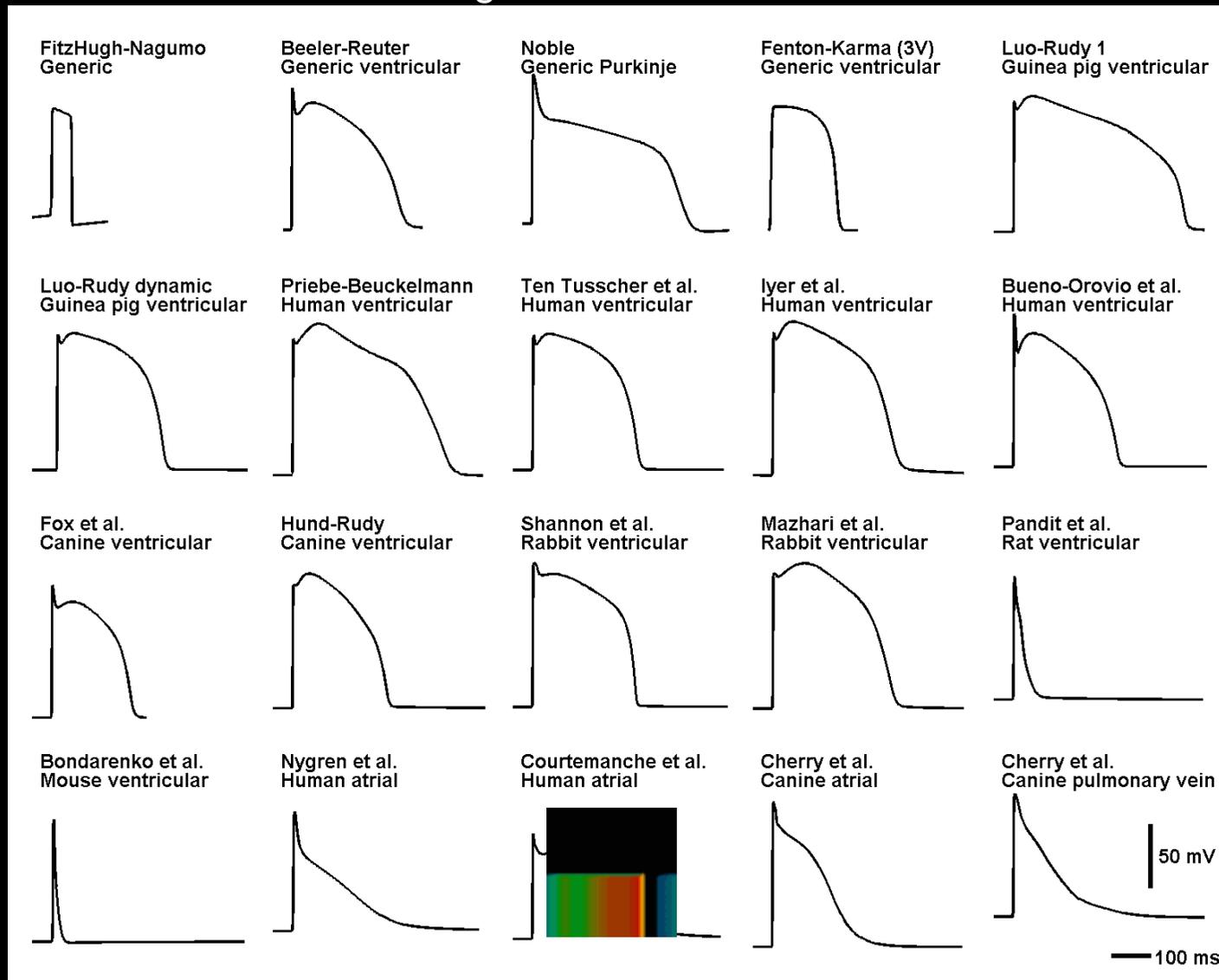


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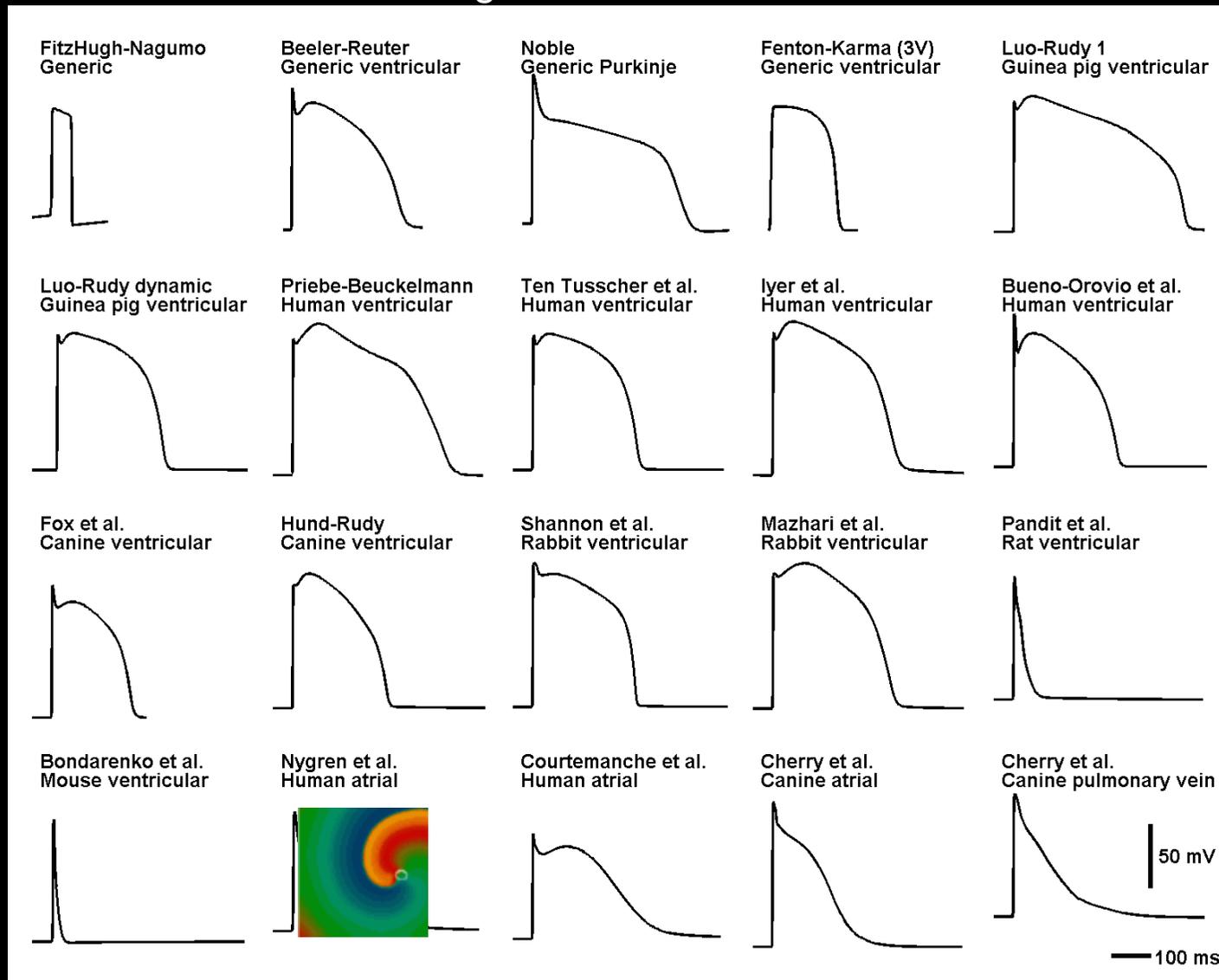


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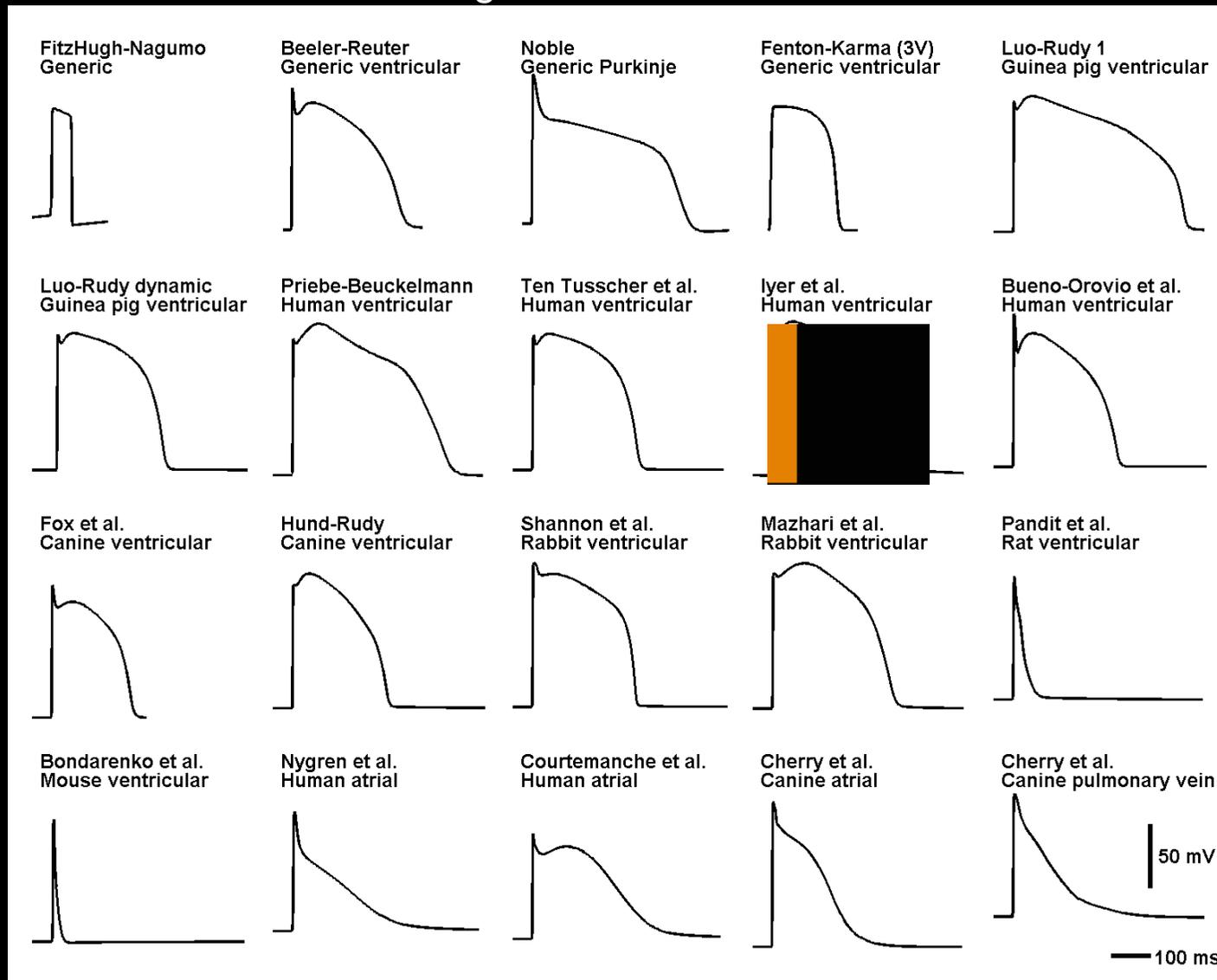


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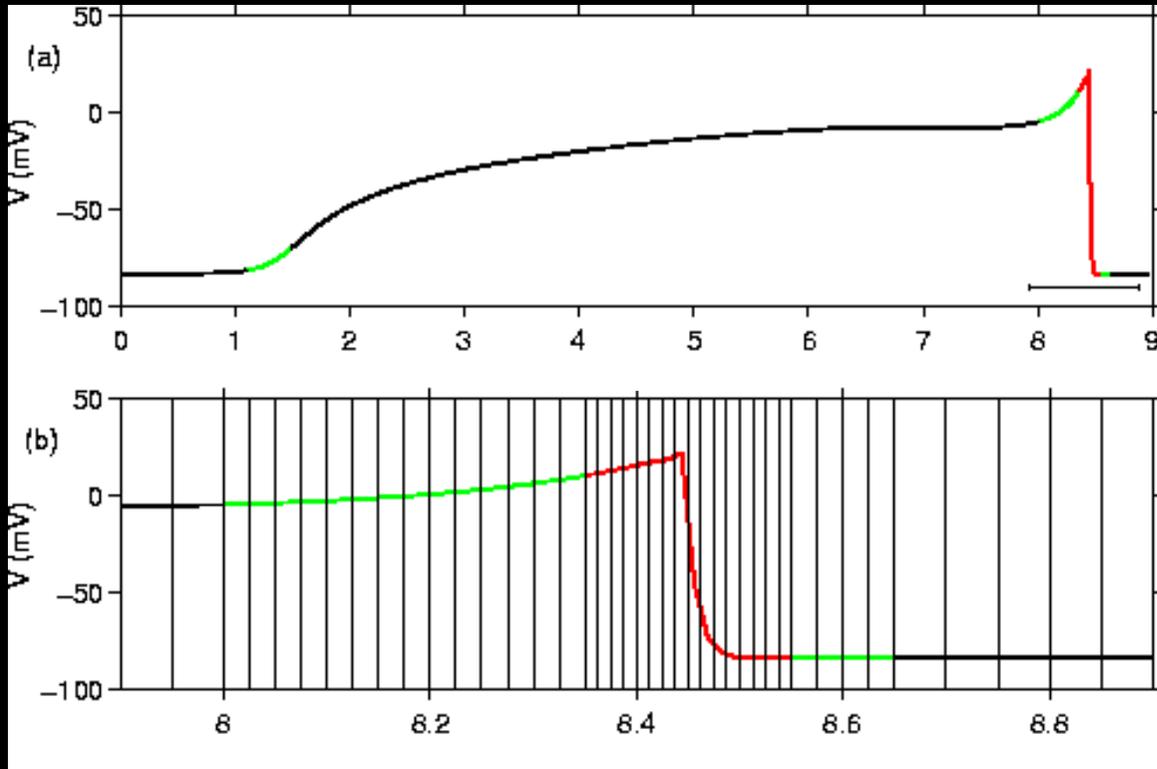


67 variables
2D (200x200x67
=2680000)

Modeling and simulations challenges

- Stiff ODEs ($dt \sim 0.01ms$)

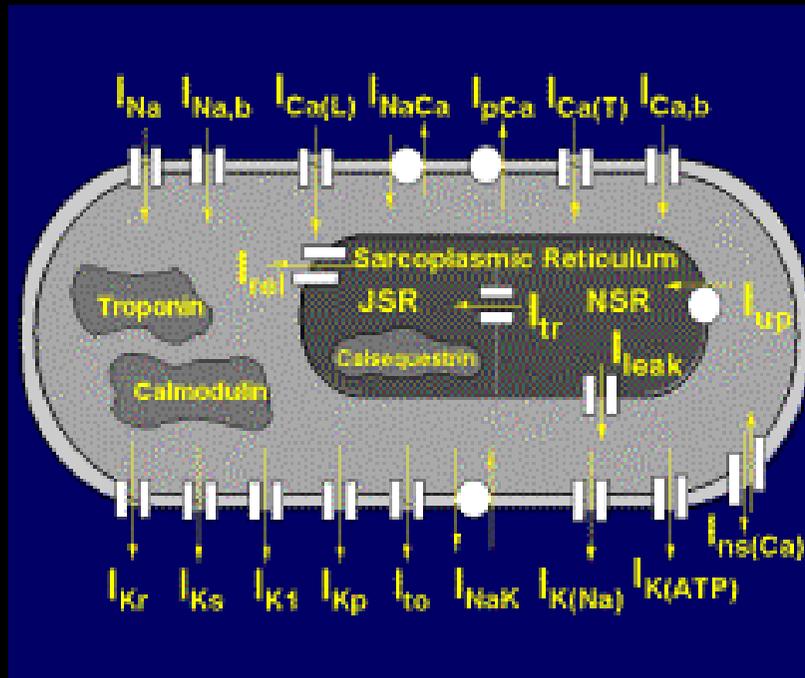
← 200ms → 4ms ↔



Adaptive mesh refinement or semi implicit methods

Modeling and simulations challenges

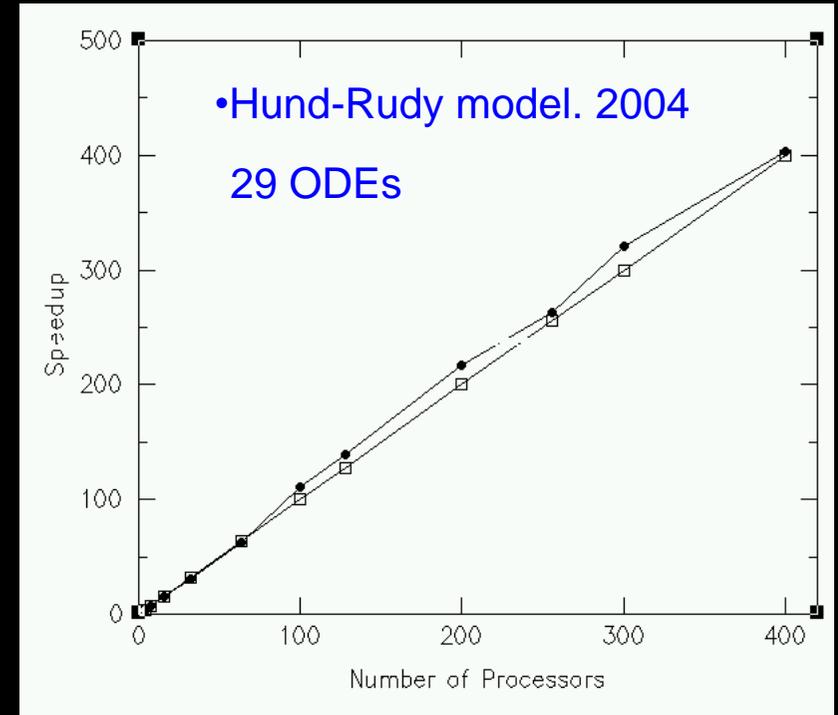
- Stiff ODEs ($dt \sim 0.01ms$)
- Number of ODEs per cell
~(4 – 100)



[4 --100]
depending on the model

Modeling and simulations challenges

- Stiff ODEs (dt ~ 0.01ms)
- Number of ODEs per cell
~(4 – 100)
- Number of cells simulated



~200 microns minimum spatial resolution

simulation on a 4cmx4cm (200x200) x 4 = 160,000 ODE equations
100 = 4,000,000 ODE equations

GPU instead of CPU

Hund-Rudy model (29 ODEs) 1 second of simulation in 2d

1,160,000 x 100,000 = 1.16×10^{11} ODE equations

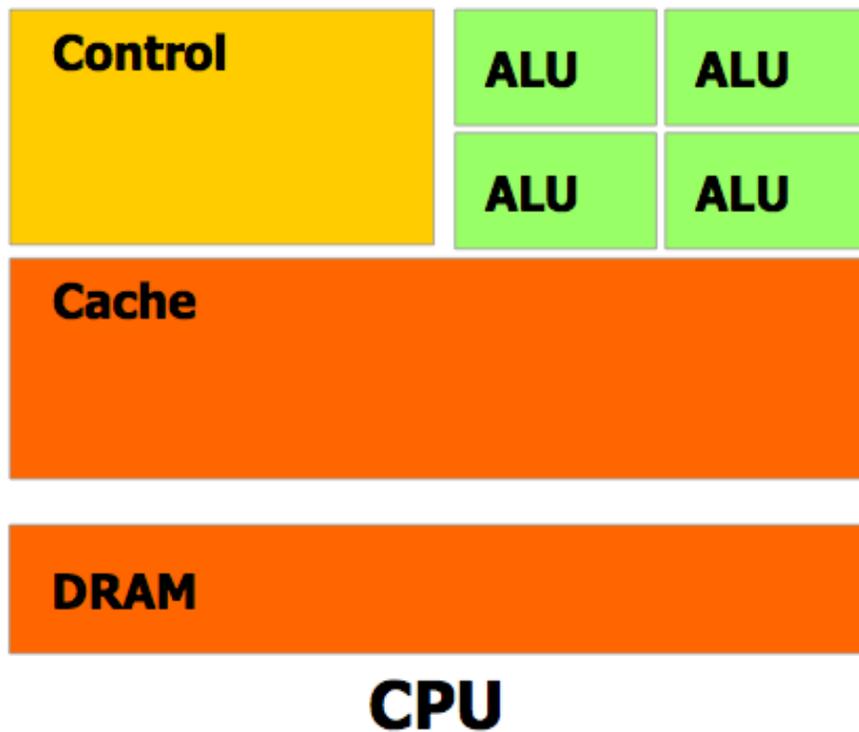
1 second of simulation in 3d : ~ twenty trillion ODE equations

GPU simulations

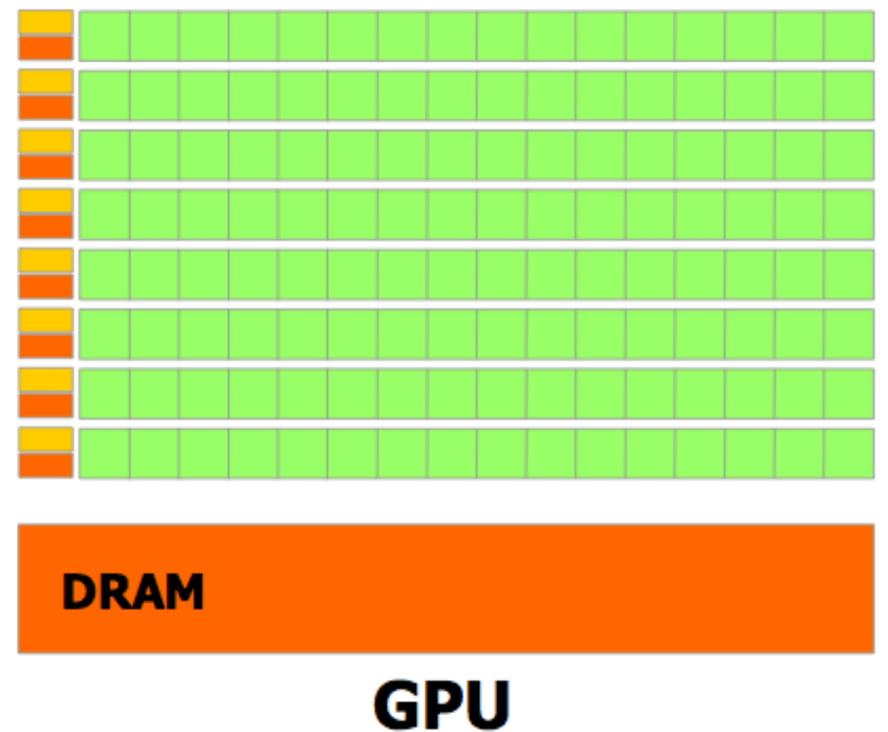
GPUs (Graphics Processing Units) have evolved far beyond single processors. Modern NVIDIA GPUs are not single processors but rather are parallel supercomputers on a chip that consist of very many, very fast processors. Contemporary NVIDIA GPUs range from 16 to 480 stream processors per card,

Available Technologies

CPU based



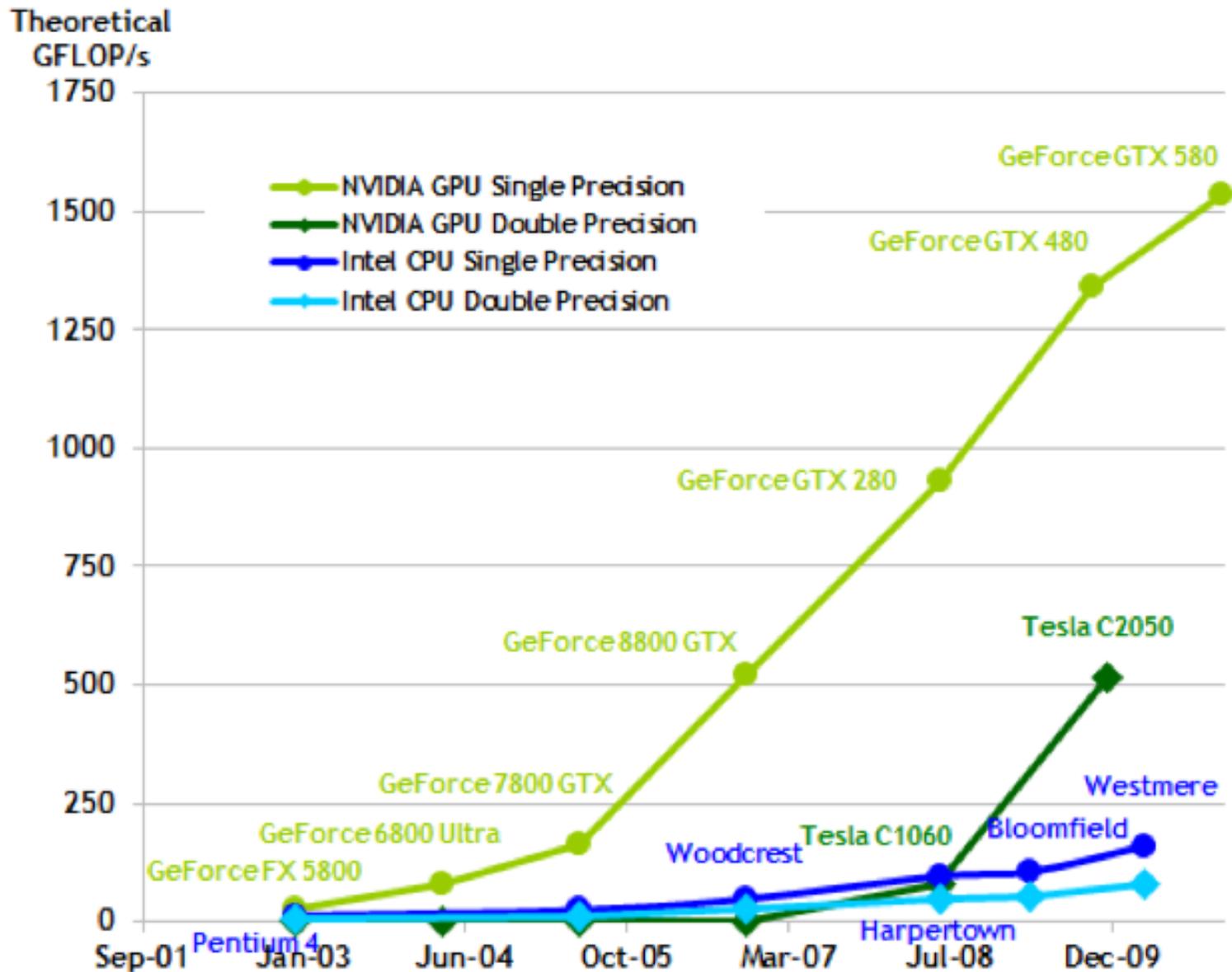
GPU based



The GPU devotes more transistors to data processing

This image is from CUDA programming guide

GPU vs CPU



Tesla C1060



30 Multiprocessors
240 Cores
Processor core clock: 1.296 GHz
933 Gigaflops (Single precision)
78 Gigaflops (Double Precision)
Max Bandwidth(102 Gigabytes/sec)
4 GB of DRAM

Cost: 1000 \$

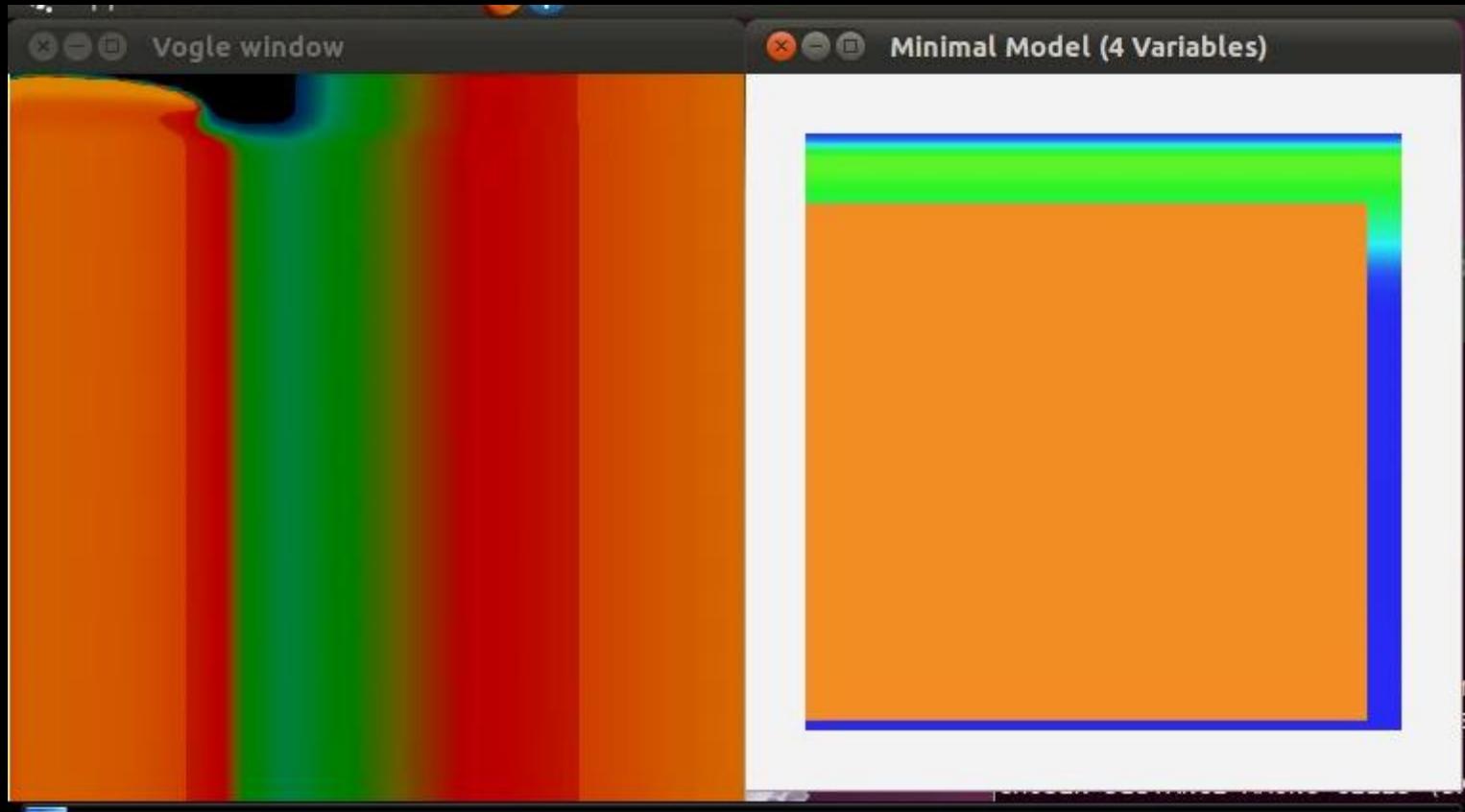
Fermi C2070



14 Multiprocessors
448 Cores
Processor core clock: 1.15 GHz
1030 Gigaflops (Single precision)
515 Gigaflops (Double precision)
Max Bandwidth (144 GBytes/sec)
6 GB of DRAM

Cost: 3200 \$

GPU near-realtime simulations



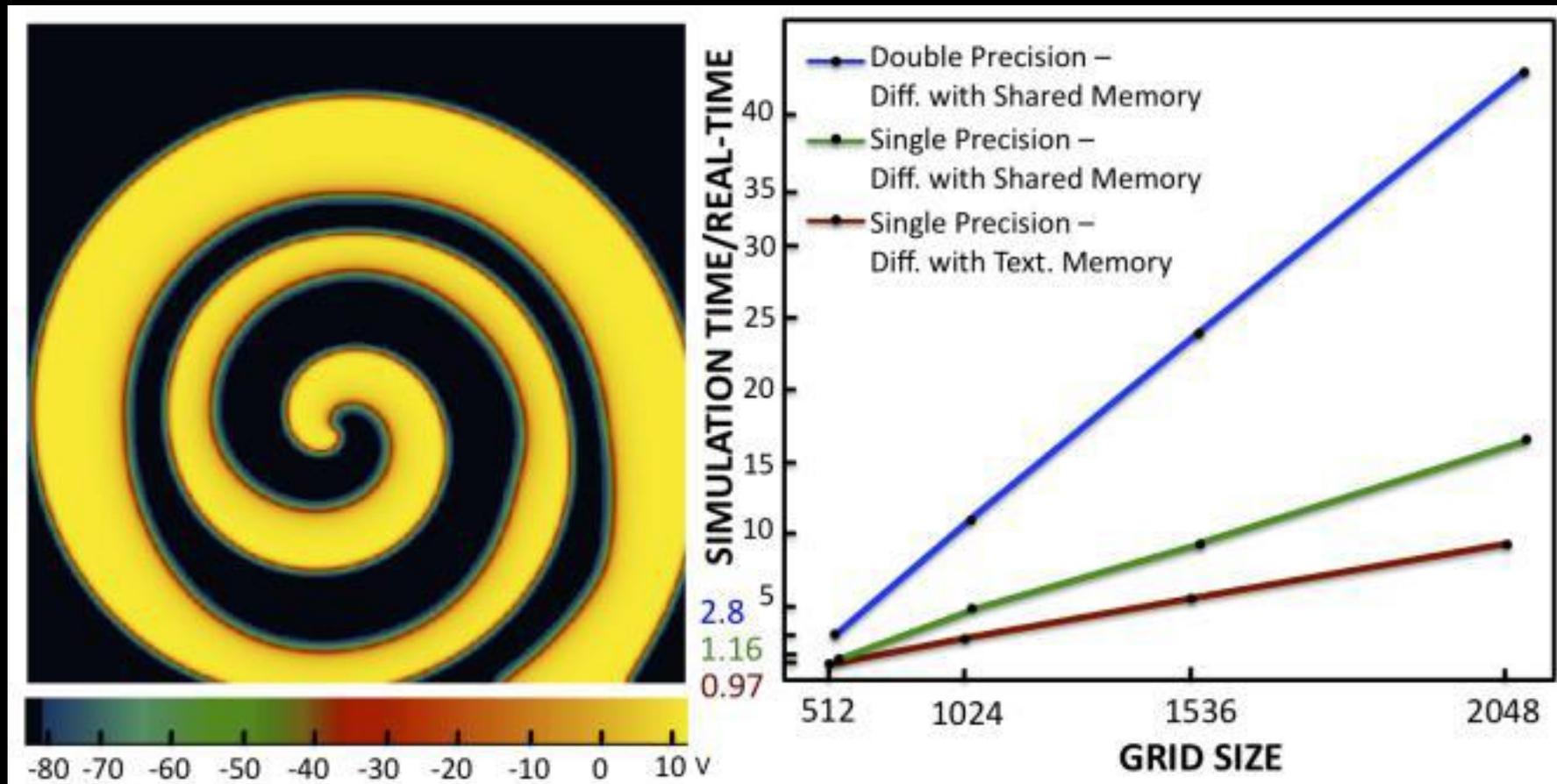
CPU

(12.8x12.8 cm²)

GPU

Same machine

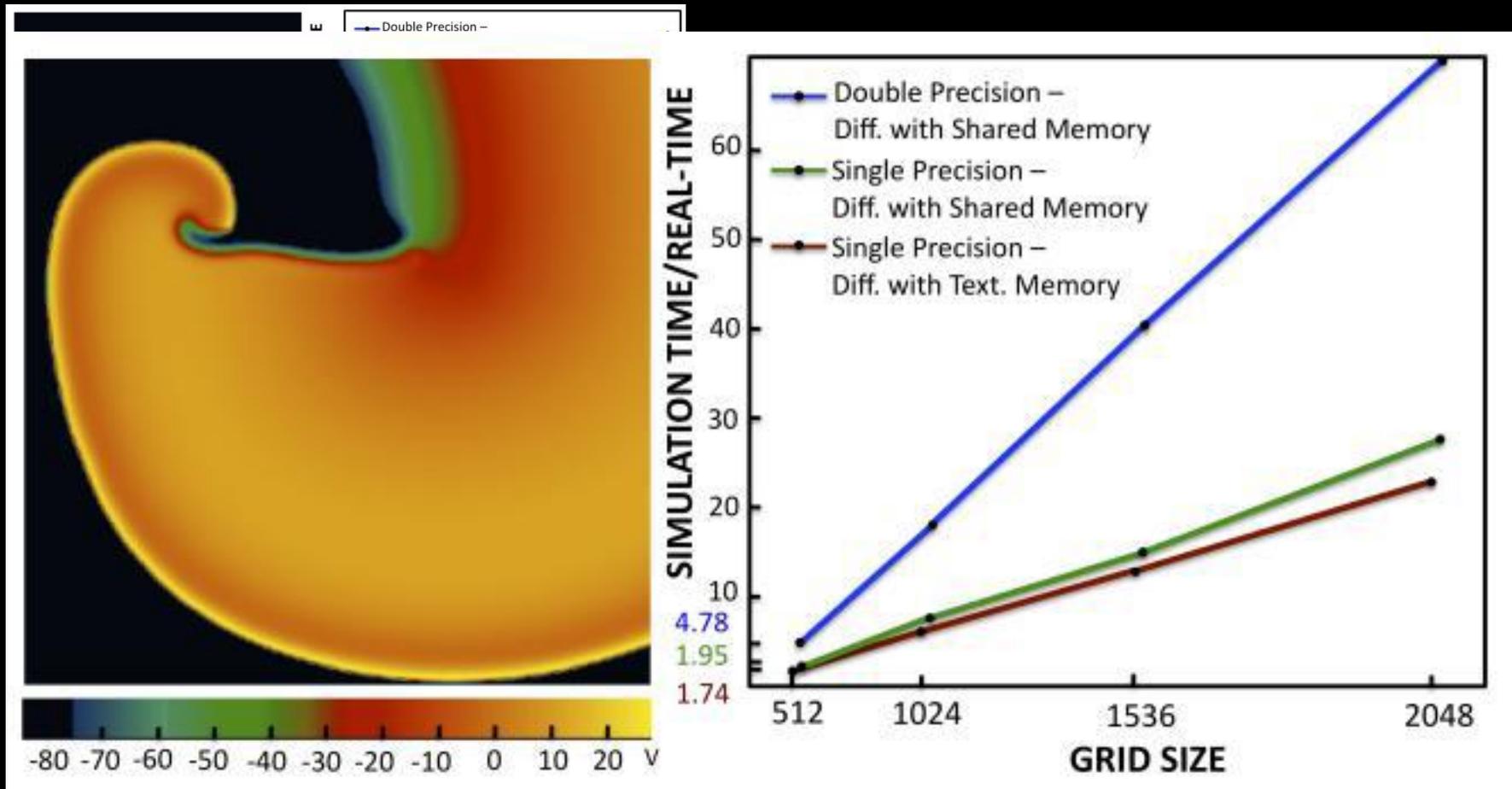
GPU near-realtime simulations



2^{18} (12.8x12.8 cm²) \rightarrow 2^{22} : ~4.2million nodes

2V Karma model

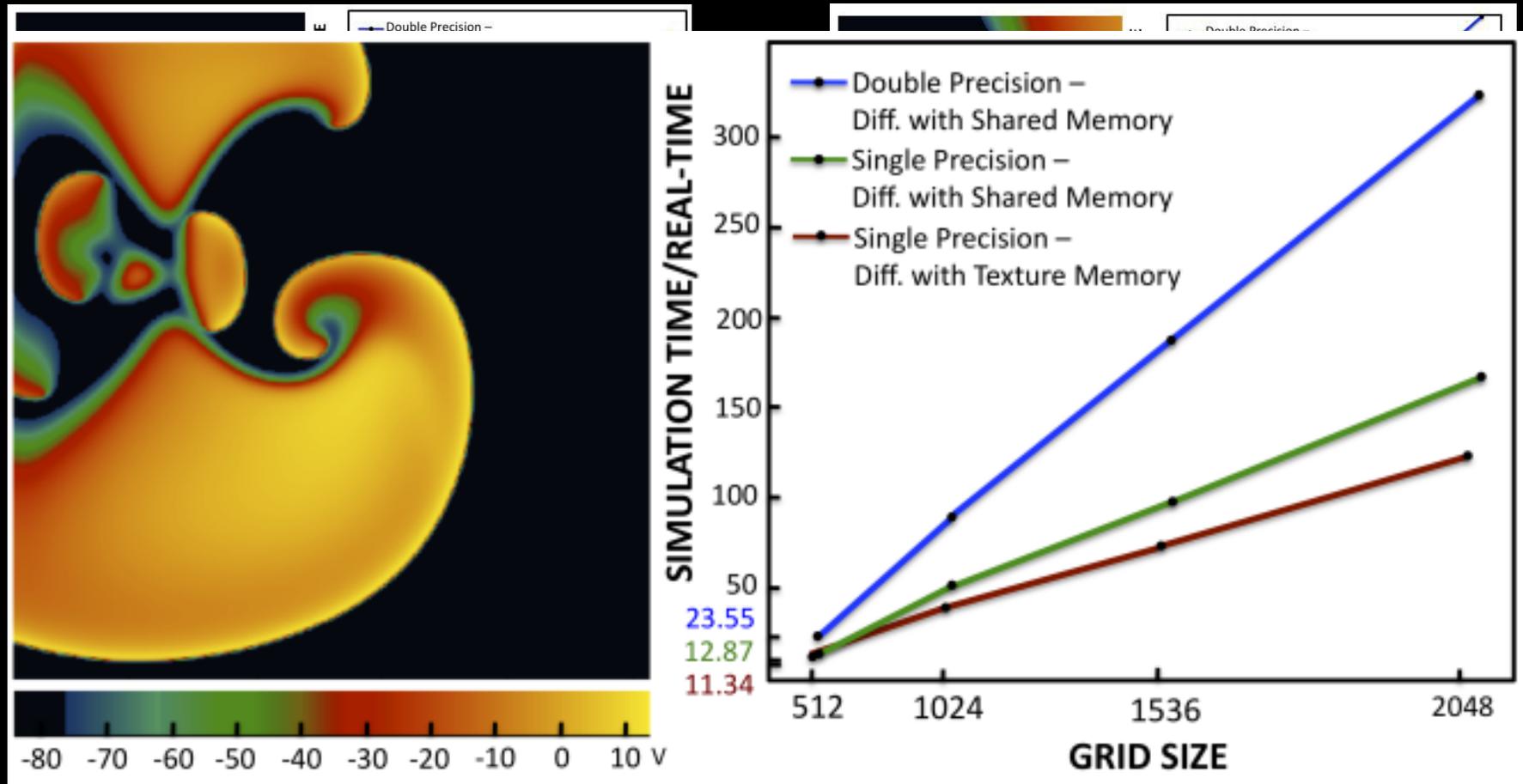
GPU near-realtime simulations



4V Minimal model

2^{18} (12.8x12.8 cm²) \rightarrow 2^{22} : ~4.2million nodes

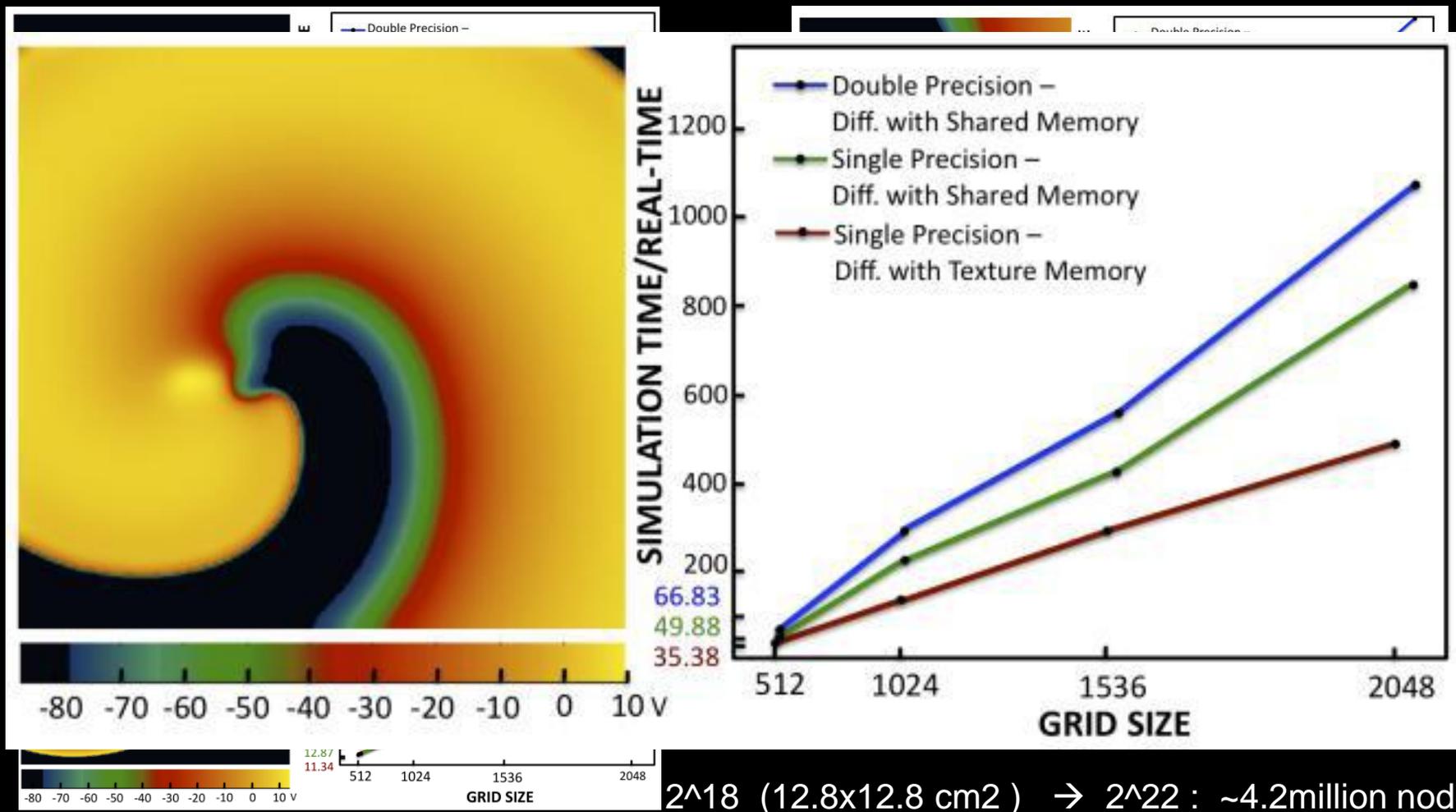
GPU near-realtime simulations



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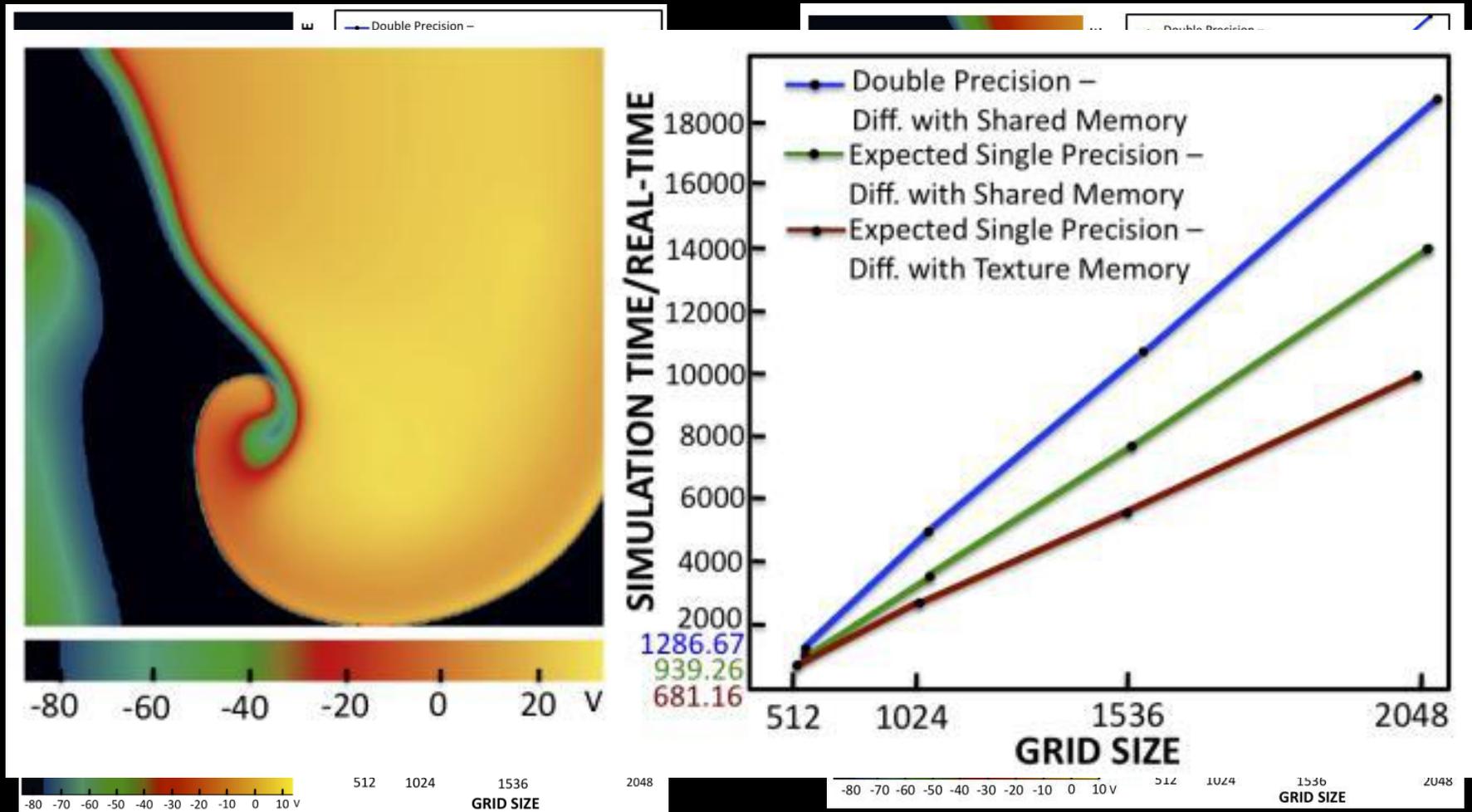
8V BR model

GPU near-realtime simulations



19V TP model

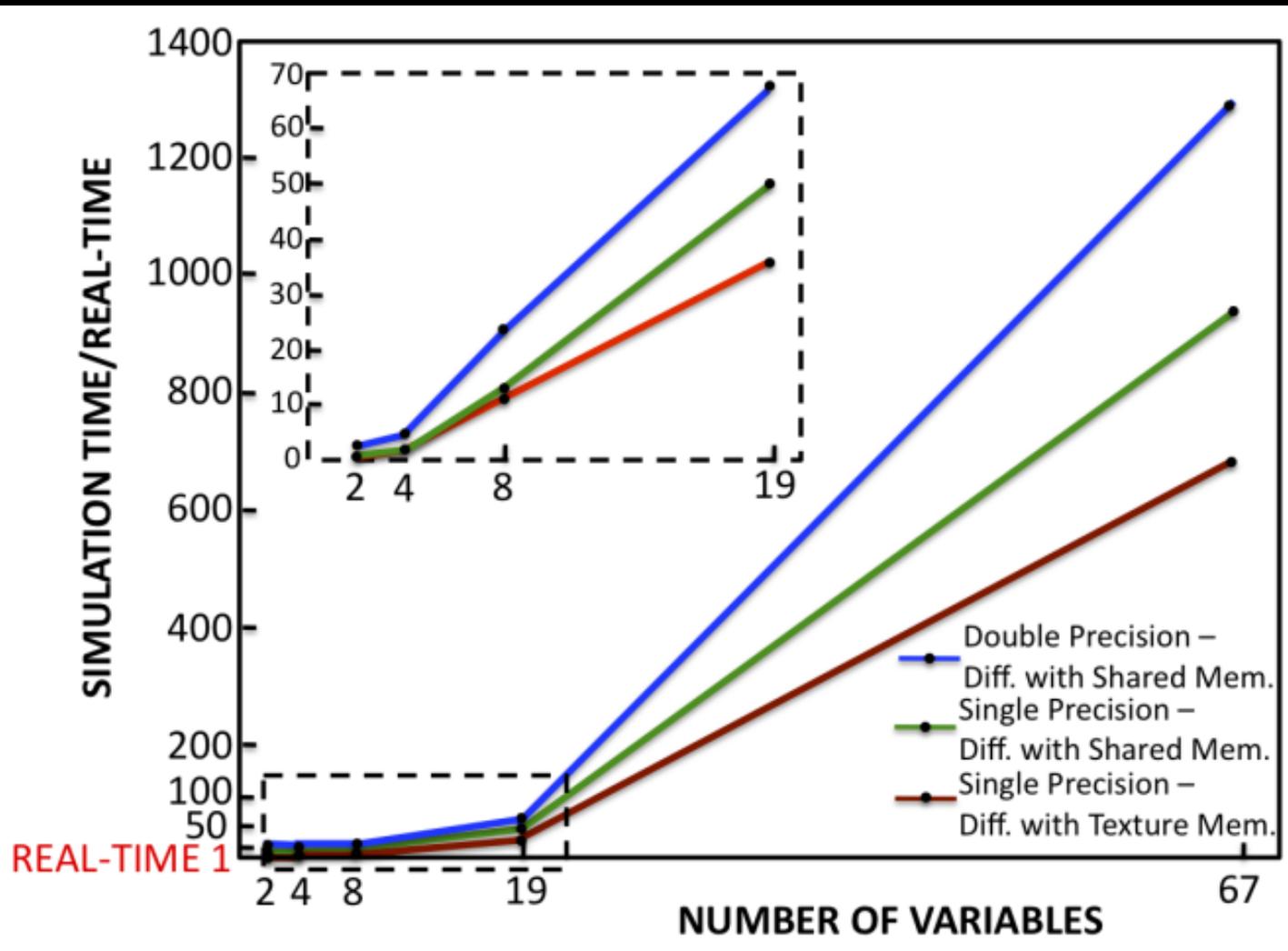
GPU near-realtime simulations



67V Iyer et al, model

2^{18} (12.8x12.8 cm²) \rightarrow 2^{22} : ~4.2million nodes

GPU near-realtime simulations



WebGL + HTML5

- **WebGL** (*Web Graphics Library*) uses **JavaScript, Shader language and OpenGL** for interactive simulations within any compatible web browser without the use of plug-ins.

Browsers supporting WebGL: Google Chrome
Fire Fox, Mozilla ([permission](#))

Examples of reaction diffusion models with WebGL in 2D

- Multiple spiral waves (chaos). [2V Barkley model](#)
- Spiral wave (2V Karma model) [interaction with the code](#)
- 1962 Noble Model ([tip trajectory and breakup](#))

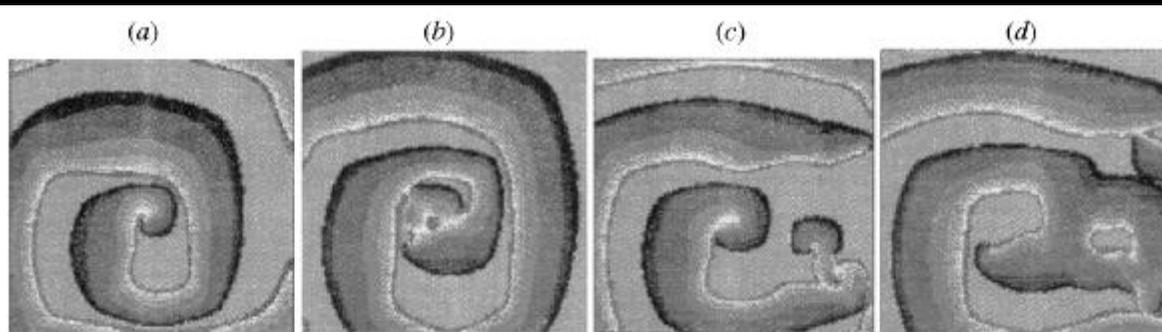


Figure 1. Spiral breakup in a Noble (1962) model of cardiac tissue. Reproduced from Panfilov & Holden (1990) with permission from Elsevier Science. Sequential snapshots of potential distribution: (a) $t = 1720$ ms; (b) $t = 1880$ ms; (c) $t = 2920$ ms; (d) $t = 3080$ ms. Potentials are coded in equal 10 mV steps, from -85 mV (dark) to $+5$ mV (white).

Examples of reaction diffusion models with WebGL in 2D

- Minimal model of Human AP
- Beeler Reuter model (1977) Super computer Winfree 1991
- 3D simulations, (scroll waves, breakup in 3d)
- GPU for other CMACS problems?