Towards Curvature-Based Prediction of Spiral Breakup in Cardiac Tissue

Abhishek Murthy

Stony Brook University (SBU)

amurthy@cs.sunysb.edu

Joint Work with Ezio Bartocci, Prof. Radu Grosu and Prof. Scott Smolka

April 28, 2011

Outline

1. **Introduction**

Outline

Outline

1. **Introduction**
2. **Wave Breaks and Atrial Fibrillation**
3. **Curvature of Cardiac Excitation Waves**

Outline

1. **Introduction**
2. **Wave Breaks and Atrial Fibrillation**
3. **Curvature of Cardiac Excitation Waves**
4. **Case Studies**

Outline

Why Atrial Fibrillation Matters

- Atrial Fibrillation (AF) - the quivering of heart muscles of atrial chambers, is the most common cardiac arrhythmia.
- Prevalent in 2.66 Million Americans, AF responsible for 14,490 deaths in 2010.
- As an independent risk factor for ischemic strokes, responsible for at least 15% to 20% cases.

Cardiac Excitation Waves

- Modelling electrical excitation of cardiac tissue as a reaction-diffusion system - Minimal Model
- Simulating model under Isotropic Diffusion (ID)

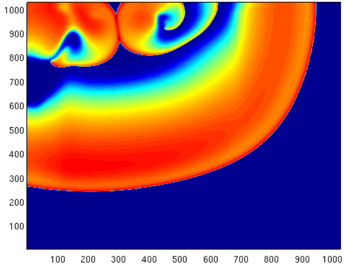


Figure: One time step of simulating cardiac electrical conduction under ID

Wave Breaks and AF

- Spatio-temporal description of the fibrillating cardiac tissue involves wave breaks or phase singularities.
- Curved waves break up near regions of high curvature.

(Loading breakupExample.avi)

Wave Breaks and AF

- Spatio-temporal description of the fibrillating cardiac tissue involves wave breaks or phase singularities.
- Curved waves break up near regions of high curvature.

(Loading breakupExample.avi)

- Predicting wave break-ups will help predict the onset of AF.

Wave Breaks and AF - A closer look

- Wave propagation speed and curvature are related.

Wave Breaks and AF - A closer look

- Wave propagation speed and curvature are related.
- If V is propagation speed and K, the curvature, then

$$V(K) = V_0 - DK$$

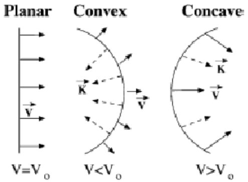
V_0 = speed of a planar wave D = diffusion co-efficient

Wave Breaks and AF - A closer look

- Wave propagation speed and curvature are related.
- If V is propagation speed and K, the curvature, then

$$V(K) = V_0 - DK$$

V_0 = speed of a planar wave D = diffusion co-efficient

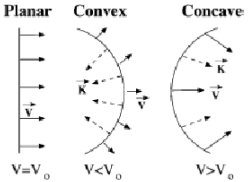


Wave Breaks and AF - A closer look

- Wave propagation speed and curvature are related.
- If V is propagation speed and K, the curvature, then

$$V(K) = V_0 - DK$$

V_0 = speed of a planar wave D = diffusion co-efficient



- Curved waves break near regions of high curvature - wave propagation velocity decreases with increasing convexity. Thus wave breaks up at critical curvature $K_{cr} = V_0/D$

Curvature of Cardiac Excitation Waves

Requirements for estimating and analysing the curvature of excitation waves (for prediction purposes):

1. Curvature estimation must be accurate.

2. Curvature should be estimated continuously along the length of the wave.

Curvature of Cardiac Excitation Waves

Requirements for estimating and analysing the curvature of excitation waves (for prediction purposes):

1. Curvature estimation must be accurate.
2. Curvature should be estimated continuously along the length of the wave.

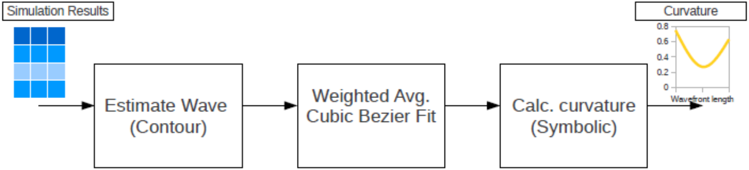


Figure: Curvature Estimation of Cardiac Excitation Waves

Curvature - Estimating Excitation Waves

- Given a simulation of a grid G of $m \times n$ cells, wave $W(c, t)$ can be written as

$$W(c, t) = \{(x, y) | x, y \in \mathbb{R} \; F(x, y) = c \; at \; time \; t\}$$

Where $F(x, y)$ = interpolation of the simulation results onto $\mathbb{R}^2$

Curvature - Estimating Excitation Waves

- Given a simulation of a grid G of $m \times n$ cells, wave $W(c,t)$ can be written as

$$W(c,t) = \{(x,y) | x, y \in \mathbb{R} \; F(x,y) = c \; at \; time \; t\}$$

  Where $F(x,y)$ = interpolation of the simulation results onto $\mathbb{R}^2$
- Check for intersection of wave and an edge of the grid.

- Given a simulation of a grid G of $m \times n$ cells, wave $W(c, t)$ can be written as

$$W(c,t) = \{(x,y)|x,y \in \mathbb{R}\ F(x,y) = c\ at\ time\ t\}$$

Where $F(x, y)$ = interpolation of the simulation results onto $\mathbb{R}^2$

- Check for intersection of wave and an edge of the grid.
- Intersection point is obtained by linear interpolation. Implemented using *contour* function of Matlab

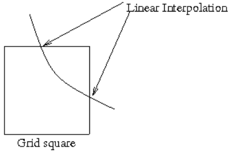


Figure: Contour estimation on grid

Curvature - Estimating Excitation Waves

- Given a simulation of a grid G of $m \times n$ cells, wave $W(c, t)$ can be written as

$$W(c,t) = \{(x,y) | x,y \in \mathbb{R}\ F(x,y) = c\ at\ time\ t\}$$

  Where $F(x, y)$ = interpolation of the simulation results onto $\mathbb{R}^2$
- Check for intersection of wave and an edge of the grid.
- Intersection point is obtained by linear interpolation. Implemented using *contour* function of Matlab

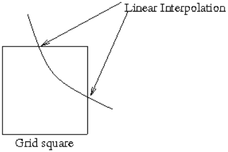


Figure: Contour estimation on grid

- Track the same wave across different time steps of the simulation.

Curvature - Cubic Bézier Fits of Waves

- Obtain C2 continuous Bézier fit that can be used for symbolic curvature estimation.

Curvature - Cubic Bézier Fits of Waves

- Obtain C2 continuous Bézier fit that can be used for symbolic curvature estimation.
- Fit each of the overlapping strip with cubic Bézier curves of the form:

$$X_j(t) = (1-t)^3 P_j^0 + 3t(1-t)^2 P_j^1 + 3t^2(1-t) P_j^2 + t^3 P_j^3. \ t \in [0,1] \quad (1)$$

$$Y_j(t) = (1-t)^3 Q_j^0 + 3t(1-t)^2 Q_j^1 + 3t^2(1-t) Q_j^2 + t^3 Q_j^3. \ t \in [0,1] \quad (2)$$

where j is the strip index.

Curvature - Cubic Bézier Fits of Waves

- Obtain C2 continuous Bézier fit that can be used for symbolic curvature estimation.
- Fit each of the overlapping strip with cubic Bézier curves of the form:

$$X_j(t) = (1-t)^3 P_j^0 + 3t(1-t)^2 P_j^1 + 3t^2(1-t) P_j^2 + t^3 P_j^3. \; t \in [0,1] \quad (1)$$

$$Y_j(t) = (1-t)^3 Q_j^0 + 3t(1-t)^2 Q_j^1 + 3t^2(1-t) Q_j^2 + t^3 Q_j^3. \; t \in [0,1] \quad (2)$$

where j is the strip index.

- In the region of overlap take weighted average of the two curves.

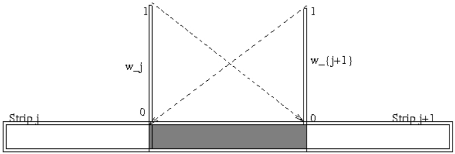


Figure: Weighted average based Bézier curve fitting

Curvature - Symbolic Curvature Estimation

- Closed form expressions corresponding to C2 smooth Bézier fit can be processed symbolically in MATLAB's symbolic computation toolbox.

Curvature - Symbolic Curvature Estimation

- Closed form expressions corresponding to C2 smooth Bézier fit can be processed symbolically in MATLAB's symbolic computation toolbox.
- If $X_j(t)$ and $Y_j(t)$ denote the fit for a strip, then curvature is calculated as

$$\kappa_j(t) = \frac{|r_j'(t) \times r_j''(t)|}{|r_j'(t)|^3} \tag{3}$$

where $r_j(t) = [X_j(t), Y_j(t)]$ is the position vector described by the Bézier curve.

- Closed form expressions corresponding to C2 smooth Bézier fit can be processed symbolically in MATLAB's symbolic computation toolbox.
- If $X_j(t)$ and $Y_j(t)$ denote the fit for a strip, then curvature is calculated as

$$\kappa_j(t) = \frac{|r'_j(t) \times r''_j(t)|}{|r'_j(t)|^3} \tag{3}$$

  where $r_j(t) = [X_j(t), Y_j(t)]$ is the position vector described by the Bézier curve.
- Continuous closed form of $\kappa_j(t) => $ continuous curvature estimate along wavefront

Curvature Estimation of Cardiac Excitation Waves - Example

(Loading circularCoreCurvatureExample.avi)

Case Studies

1. Linear core generated with Minimal Model (1024x1024)
2. Spiral Breakup generated with Beeler Reauter Model(1024x1024)

Linear Core

(Loading linearCoreCurvature.avi)

Linear Core - Curvature Trend

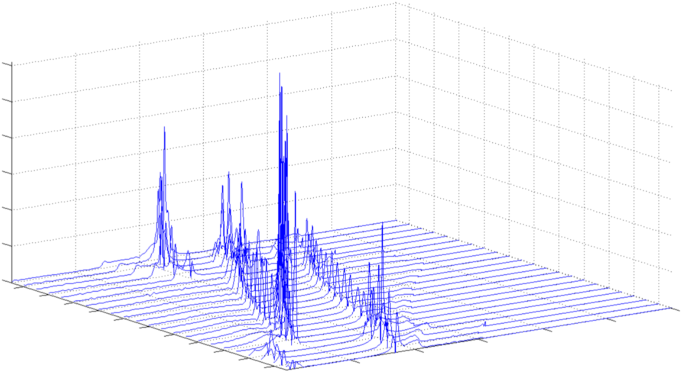


Figure: Curvature trend for linear core till first turn

Wave Breakup

(Loading brkupCurvature.avi)

Future Work

- Collect training data by simulating different wave break scenarios.
- Learn patterns of wave break-ups based up morphological features.
- Predict the temporal behaviour using the patterns learnt.

Overview

1. Predicting wave-breaks for predicting AF

Overview

1. Predicting wave-breaks for predicting AF
2. Curvature and spatio-temporal behaviour of cardiac waves are related

Overview

1. Predicting wave-breaks for predicting AF
2. Curvature and spatio-temporal behaviour of cardiac waves are related
3. Requirements for curvature estimation

Overview

1. Predicting wave-breaks for predicting AF
2. Curvature and spatio-temporal behaviour of cardiac waves are related
3. Requirements for curvature estimation
4. Weighted average based fitting with cubic Bézier curves

Overview

1. Predicting wave-breaks for predicting AF
2. Curvature and spatio-temporal behaviour of cardiac waves are related
3. Requirements for curvature estimation
4. Weighted average based fitting with cubic Bézier curves
5. Symbolic curvature calculation

Overview

1. Predicting wave-breaks for predicting AF
2. Curvature and spatio-temporal behaviour of cardiac waves are related
3. Requirements for curvature estimation
4. Weighted average based fitting with cubic Bézier curves
5. Symbolic curvature calculation
6. Case studies show potential of using curvature to analyse cardiac waves.

References

1. "Heart Disease and Stroke Statistics 2011 update: A Report from the American Heart Association", Véronique L. Roger et. al. Circulation Journal of the American Heart Association.
2. "Measuring Curvature and Velocity Vector Fields for Waves of Cardiac Excitation in 2-D Media", Matthew W. Kay and Richard A. Gray, IEEE Transactions on Biomedical Engineering 2005.
3. "Role of Wavefront Curvature in Propagation of Cardiac Impulse", Vladmir G. Fast and André G. Kléber, Cardiovascular Research 1997.
4. "The Fibrillating Atrial Myocardium. What can the Detection of Wave Breaks Tell Us?", André G. Kléber, Cardiovascular Research 2000.

Consider an edge e on the grid G whose end points are (x_{g1}, y_{g1}) and (x_{g2}, y_{g2}) and the excitation levels at the two end points are c_1 and c_2. The wavefront crosses this edge if $c_1 \leq c \leq c_2$.

Let (x, y) be the point at which the wavefront intersects this edge. x and y can be calculated using linear interpolation as follows:

$$x = x_{g1} + \frac{c - c_1}{c_2 - c_1}(x_{g2} - x_{g1})$$

$$y = y_{g1} + \frac{c - c_1}{c_2 - c_1}(y_{g2} - y_{g1})$$

Running time for n x n grid = $O(n^2)$

Details - Bezier Curve fitting

Bezier curve:

$$X_j(t) = (1-t)^3 P_j^0 + 3t(1-t)^2 P_j^1 + 3t^2(1-t)P_j^2 + t^3 P_j^3. \ t \in [0,1] \quad (4)$$

$$Y_j(t) = (1-t)^3 Q_j^0 + 3t(1-t)^2 Q_j^1 + 3t^2(1-t)Q_j^2 + t^3 Q_j^3. \ t \in [0,1] \quad (5)$$

Error functions

$$E_x = \sum_{i=1}^{SL} [x_i - X_j(t_i)]^2 \quad (6)$$

$$E_y = \sum_{i=1}^{SL} [y_i - Y_j(t_i)]^2 \quad (7)$$

which give

$$E_x = \sum_{i=1}^{SL} [x_i - (1-t_i)^3 P_j^0 + 3t_(1-t_i)^2 P_j^1 + 3t_i^2(1-t_i)P_j^2 + t_i^3 P_j^3]^2$$

$$E_y = \sum^{SL} [y_i - (1-t_i)^3 Q_j^0 + 3t_i(1-t_i)^2 Q_j^1 + 3t_i^2(1-t_i)Q_j^2 + t_i^3 Q_j^3]^2$$

P_j^1 and P_j^2 can be obtained at the minimum value of E_x by

$$\frac{\partial E_x}{\partial P_j^1} = 0$$

$$\frac{\partial E_x}{\partial P_j^2} = 0$$

Solving the above two equations we obtain the following expressions for P_j^1 and P_j^2:

$$P_j^1 = \frac{\alpha_2^j \beta_1^j - \alpha_3^j \beta_2^j}{\alpha_1^j \alpha_2^j - {\alpha_3^j}^2} \quad (8)$$

$$P_j^2 = \frac{\alpha_1^j \beta_2^j - \alpha_3^j \beta_1^j}{\alpha_1^j \alpha_2^j - \alpha_3^{2^j}} \quad (9)$$

Details - Bezier Curve fitting

where $\alpha_1, \alpha_2, \alpha_3, \beta_1$ and β_2 for each segment are given by:

$$\alpha_1 = 9 \sum_{i=1}^{SL} [t_i^2 (1 - t_i)^4]$$

$$\alpha_2 = 9 \sum_{i=1}^{SL} [t_i^4 (1 - t_i)^2]$$

$$\alpha_3 = 9 \sum_{i=1}^{SL} [t_i^3 (1 - t_i)^3]$$

$$\beta_1 = 3 \sum_{i=1}^{SL} [t_i (x_i - (1 - t_i)^3 P_0 - t_i^3 P_3)(1 - t_i)^2]$$

$$\beta_2 = 3 \sum_{i=1}^{SL} [t_i^2 (x_i - (1 - t_i)^3 P_0 - t_i^3 P_3)(1 - t_i)]$$