Towards a Theory of Composition for Distributed Control

Future Work

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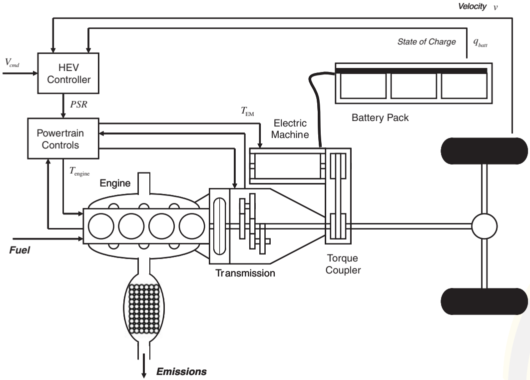
Motivation



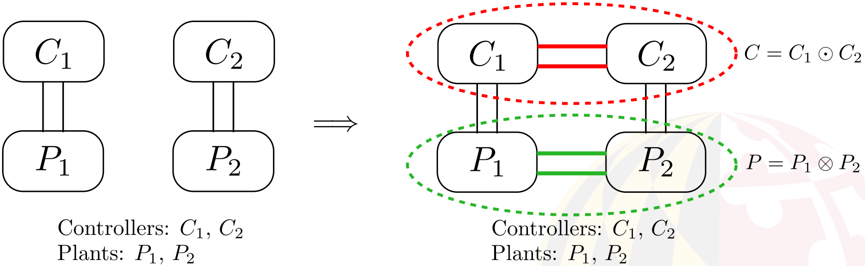
(a) Powertrain Diagram of a Hybrid Truck [Tate]

(b) On the road! [Wikipedia]

- *Notice the presence of multiple controllers for separate subsystems (in particular for the engine)!*

Composition of Controlled Systems

- What happens when we connect multiple controlled systems?



- What happens at the interface? What if both controllers want to use the same actuator?
- Do the composed controllers still “control” the composed plant?
- What properties are preserved under these operations?

Previous Work

Previous work from computer scientists: (e.g. [Bornot] and [Henzinger])

- Composition mostly on the discrete side
- Mostly concerned with linear hybrid automata
- Models tend to use $\dot{x} = A_j x$ instead of, e.g. $\dot{x} = \widetilde{A}_j x + \widetilde{B}_j f_j(x)$ (where j indexes states in an automaton)

Important goal 1

- A more general notion of composition is needed

Previous Work (continued)

Previous work in the control community:

- This is not distributed control (in the usual sense)
- LOTS of work on input/output structures, e.g. feedback
- Behavioral approach typically applied to a single controller/plant (e.g. [van der Schaft 04], [van der Schaft 02], [Julius], [Tabuada])

Important goal 2

- Treat composition from a component based perspective

For both goals, we need to be able to think more generally about the composition of continuous systems.

Behavioral Approach [Willems 07]

Model dynamical systems in terms of "behaviors", i.e. time trajectories of variables. Compare to the language of an automaton.

Definition [Willems 07]

A **Dynamical System** Σ is a triple:

$$\Sigma = (\mathbb{T}, \mathbb{W}, \mathcal{B})$$

where

$$\mathbb{T} \triangleq \text{time axis (e.g. } \mathbb{R} \text{ for time).}$$

$$\mathbb{W} \triangleq \text{signal space (e.g. } \mathbb{R}^n \text{ for } n \text{ real signals)}$$

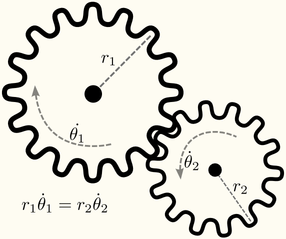
$$\mathcal{B} \triangleq \text{set of behaviors} \subseteq \mathbb{W}^{\mathbb{T}} \text{ (i.e. maps from } \mathbb{T} \text{ to } \mathbb{W}\text{)}$$

The Behavioral Approach (continued)

The behavioral approach of Willems provides a physically sound means of *interconnecting dynamical systems* through the idea of *shared variables*.

Example

- Σ_1 might model an electric motor
  Σ_2 might model a transmission
- Connect motor to the transmission with a gear $\implies$ (linear) velocities are now shared!



- (Notice input/output ambiguity under regenerative braking!)

Notation

Let $\Sigma_i = (\mathbb{T}, \mathbb{W}_i, \mathcal{B}_i)$, $i \in \{1, 2\}$ be two dynamical systems where:

$$\mathbb{T} = \mathbb{R}$$

$$\mathbb{W}_i = \mathbb{R}^{n_i} = \mathbb{X}_{i,1} \times \ldots \times \mathbb{X}_{i,n_i}$$

$$x_i \in \mathcal{B}_i \implies x_i(t) = [x_{i,1}(t) \quad \ldots \quad x_{i,n_i}(t)] \in \mathbb{W}_i \quad \forall t \in \mathbb{T}$$

Interconnection via Shared Variables [Willems 07], [Willems 97]

We can define the interconnection of Σ_1 and Σ_2 on $\mathbb{X}_{1,1}$ and $\mathbb{X}_{2,1}$ (for example) as the following dynamical system:

$$\Sigma = (\mathbb{T}, \mathbb{W}_1 \times \mathbb{W}_2, \mathcal{B})$$

where

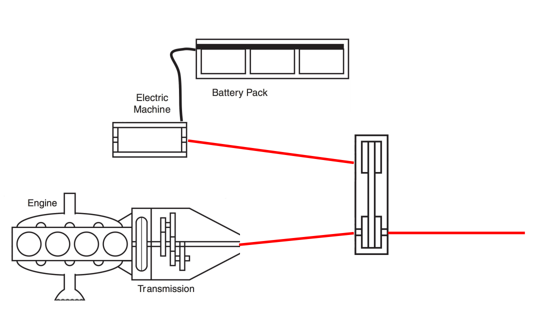
$$\mathcal{B} = \{(x_1, x_2) \in \mathcal{B}_1 \times \mathcal{B}_2 : \; x_{1,1}(t) = x_{2,1}(t) \; \forall t \in \mathbb{T}\}$$

(Duplication of $\mathbb{X}_{1,1}$ and $\mathbb{X}_{2,1}$ for notational convenience.)

The Behavioral Approach and Composition

- This notion of interconnection is a means of composing two dynamical systems
- We can think of composition more broadly, though:

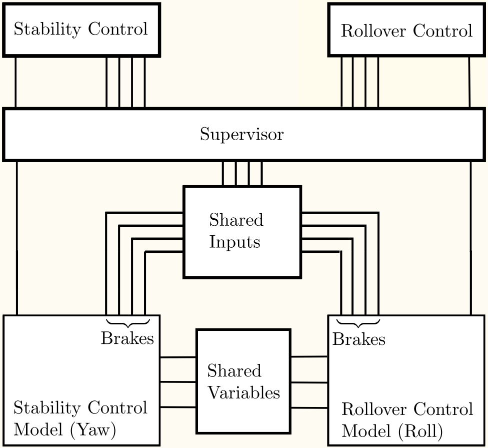
Example



Implications at the Interface

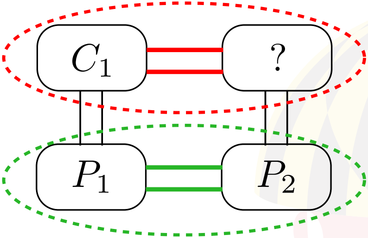
What if two controllers want to use the same actuator?

Example



Future Work

- Defining composition operators
- Invariants under composition
- Design questions, e.g.



- Stochastic systems?

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