Logic for Distributed Hybrid Systems

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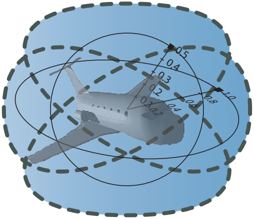


Outline

1. Motivation
2. Quantified Differential Dynamic Logic Qd$\mathcal{L}$
   - Design
   - Syntax
3. Verification of Distributed Hybrid Systems
   - Soundness and Completeness
4. Conclusions

Complex Physical Systems:

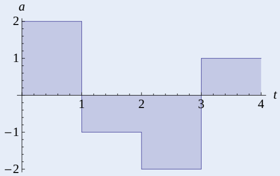
Q: I want to verify my car

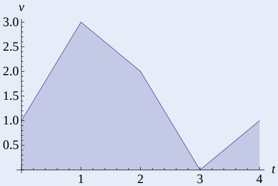
Challenge

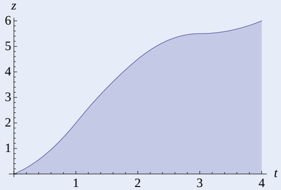
Q: I want to verify my car A: Hybrid systems

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)



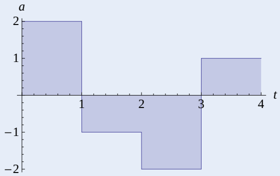


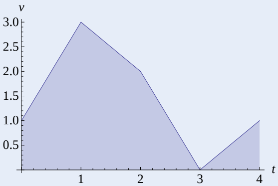


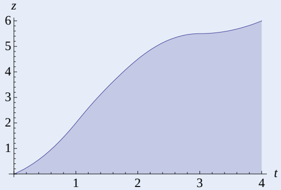
Q: I want to verify my car A: Hybrid systems Q: But there's a lot of cars!

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)



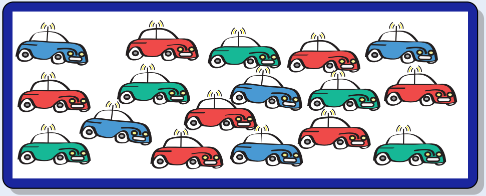




Complex Physical Systems:

Q: I want to verify a lot of cars

Challenge

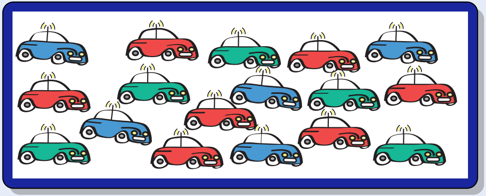


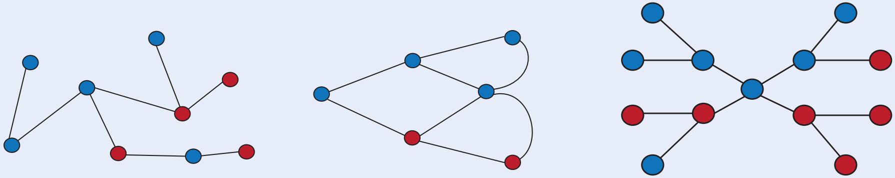
Complex Physical Systems: Distributed Systems

Q: I want to verify a lot of cars A: Distributed systems

Challenge (Distributed Systems)

- Local computation (finite state automaton)
- Remote communication (network graph)

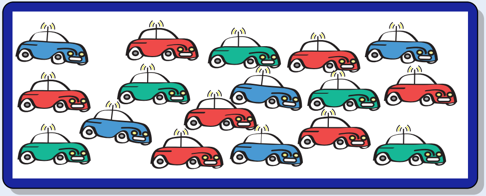


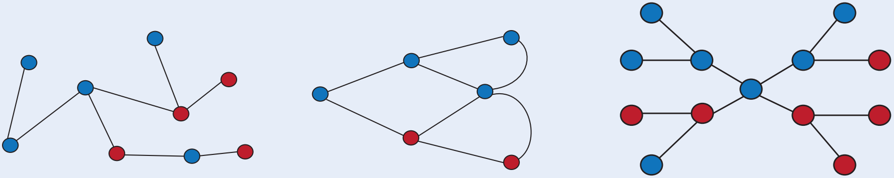


Q: I want to verify a lot of cars A: Distributed systems Q: But they move!

Challenge (Distributed Systems)

- Local computation (finite state automaton)
- Remote communication (network graph)

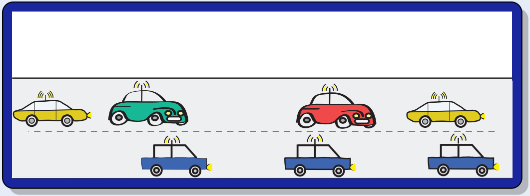




Complex Physical Systems:

Q: I want to verify lots of moving cars

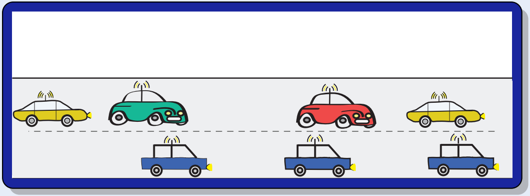
Challenge



Q: I want to verify lots of moving cars A: Distributed hybrid systems

Challenge (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)
- Structural dynamics (remote communication)



Q: I want to verify lots of moving cars A: Distributed hybrid systems

Challenge (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
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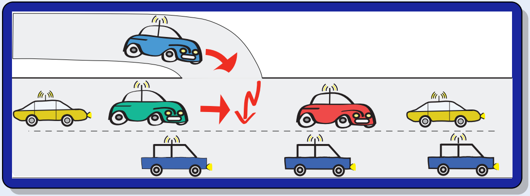
- Structural dynamics (remote communication)
- Dimensional dynamics (appearance)



Complex Physical Systems: Distributed Hybrid Systems

Q: I want to verify lots of moving cars A: Distributed hybrid systems Q: How?

Challenge (Distributed Hybrid Systems)

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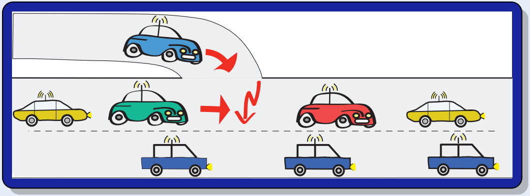
- Structural dynamics (remote communication)
- Dimensional dynamics (appearance)



State of the Art:

Shift [DGV96] The Hybrid System Simulation Programming Language

Hybrid CSP [CJR95] Semantics in Extended Duration Calculus

HyPA [CR05] Translate fragment into normal form.

χ process algebra [vBMR$^+$06] Simulation, translation of fragments to PHAVER, UPPAAL

R-Charon [KSPL06] Modeling Language for Reconfigurable Hybrid Systems

Φ-calculus [Rou04] Semantics in rich set theory

ACP_{hs}^{srt} [BM05] Modeling language proposal

OBSHS [MS06] Partial random simulation of objects

State of the Art: Modeling and Simulation

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State of the Art: Modeling and Simulation

No formal verification of distributed hybrid systems

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Contributions

1. System model and semantics for distributed hybrid systems: QHP
2. Specification and verification logic: Qd$\mathcal{L}$
3. Compositional verification for Qd$\mathcal{L}$
4. First verification approach for distributed hybrid systems
5. Sound and complete relative to differential equations
6. Verify collision freedom in a (simple) distributed car control system, where new cars may appear dynamically on the road
7. Logical foundation for analysis of distributed hybrid systems
8. Fundamental extension: first-order $x(i)$ versus primitive x

Outline

1. Motivation
2. Quantified Differential Dynamic Logic Qd$\mathcal{L}$
   - Design
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Outline (Conceptual Approach)

1. Motivation
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Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

Model (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)
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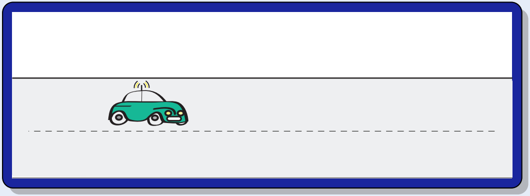

Model for Distributed Hybrid Systems

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  $x'' = a$
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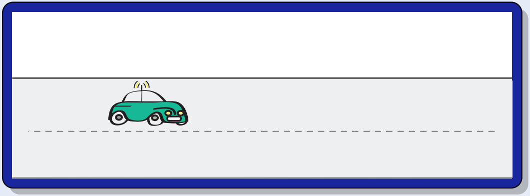

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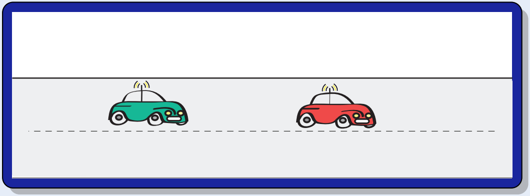
  $a := \texttt{if} .. \texttt{then} \, A \, \texttt{else} \, -b$

- Structural dynamics (remote communication)



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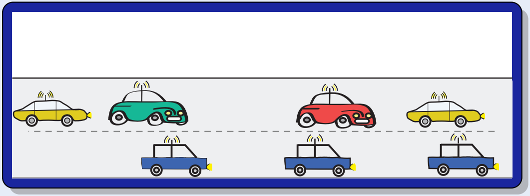

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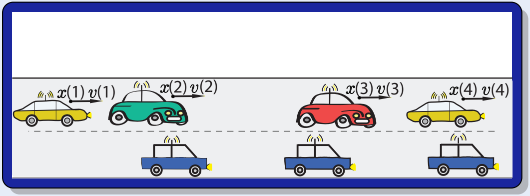

Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

Model (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
  $$x(i)'' = a(i)$$
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  $$a(i) := \texttt{if} .. \texttt{then}\, A\, \texttt{else} -b$$
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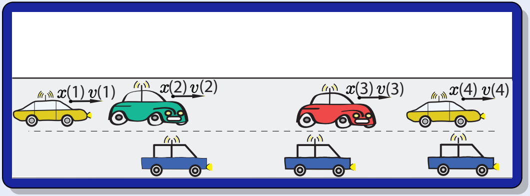

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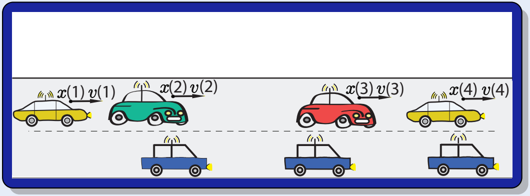

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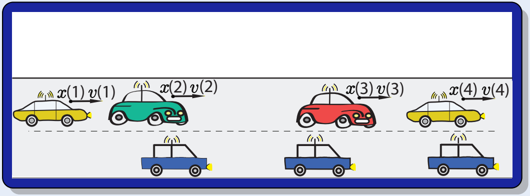
  $\ell(i) := carInFrontOf(i)$



Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems A: Quantified Hybrid Programs

Model (Distributed Hybrid Systems)

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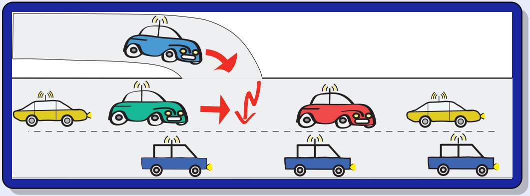

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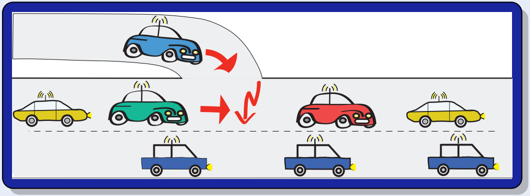
  $n := \mathtt{new}\ Car$



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

Definition (Quantified hybrid program α)

$\forall i : C\ x(s)' = \theta$	(quantified ODE)	
$\forall i : C\ x(s) := \theta$	(quantified assignment)	jump & test
$?\chi$	(conditional execution)	jump & test
$\alpha; \beta$	(seq. composition)	Kleene algebra
$\alpha \cup \beta$	(nondet. choice)	Kleene algebra
α^*	(nondet. repetition)	Kleene algebra

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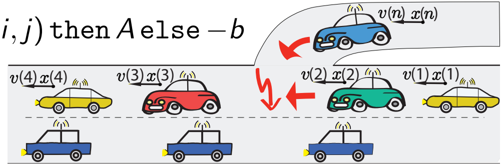
$$DCCS \equiv (ctrl; drive)^*$$

$$ctrl \equiv \forall i : C \; a(i) := \texttt{if} \; \forall j : C \; far(i,j) \; \texttt{then} \; A \; \texttt{else} \; -b$$

$$drive \equiv \forall i : C \; x(i)'' = a(i)$$



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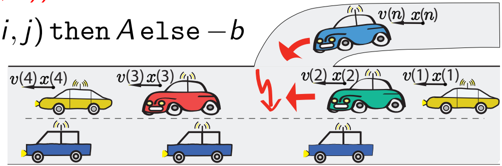

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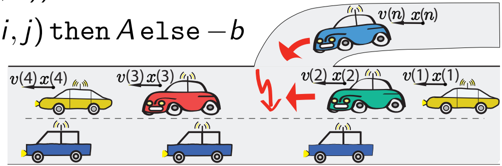
$$drive \equiv \forall i : C\ x(i)'' = a(i)$$

new C is definable!



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

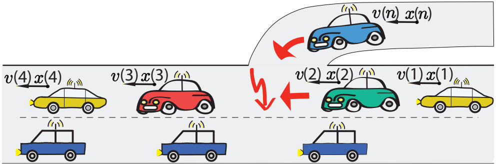
Definition (Qd$\mathcal{L}$ Formula ϕ)

$\neg, \wedge, \vee, \rightarrow, \ \forall x, \exists x, \ =, \leq, \ +, \cdot$ (ℝ-first-order part)

$[\alpha]\phi, \quad \langle\alpha\rangle\phi$ (dynamic part)

$$\forall i, j : C \ \mathit{far}(i,j) \rightarrow [(\mathit{appear}; \mathit{ctrl}; \mathit{drive})^*] \ \forall i{\neq}j : C \ x(i) \neq x(j)$$

$$\mathit{far}(i,j) \equiv i \neq j \rightarrow x(i) < x(j) \wedge v(i) \leq v(j) \wedge a(i) \leq a(j) \\ \vee x(i) > x(j) \wedge v(i) \geq v(j) \wedge a(i) \geq a(j) \ldots$$



Outline (Verification Approach)

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Soundness and Completeness

Theorem (Relative Completeness)

QdL *verification sound & complete axiomatisation of distributed hybrid systems relative to quantified differential equations.*

▸ Proof 16p.

Soundness and Completeness

Theorem (Relative Completeness)

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Corollary (Proof-theoretical Alignment)

proving distributed hybrid systems = proving dynamical systems!

Soundness and Completeness

Theorem (Relative Completeness)

Qd$\mathcal{L}$ *verification sound & complete axiomatisation of distributed hybrid systems relative to quantified differential equations.*

Proof 16p.

Corollary (Proof-theoretical Alignment)

proving distributed hybrid systems = proving dynamical systems!

Corollary (Yes, we can!)

distributed hybrid systems can be verified by recursive decomposition

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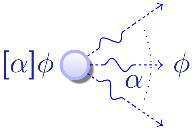
Conclusions

quantified differential dynamic logic

$\mathrm{Qd}\mathcal{L} = \mathrm{FOL} + \mathrm{DL} + \mathrm{QHP}$

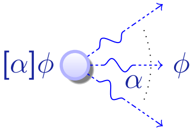


- Distributed hybrid systems everywhere
- System model and semantics
- Logic for distributed hybrid systems
- Compositional verification
- First verification approach
- Sound & complete / diff. eqn.
- Simple distributed car control verified

Conclusions

quantified differential dynamic logic

$$\mathrm{Qd}\mathcal{L} = \mathrm{FOL} + \mathrm{DL} + \mathrm{QHP}$$



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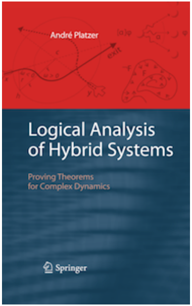

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